BYJU'S Classes

Solid State

Relation Between Radius of Atom and Edge Length and Packing Efficiency

What you already know

- Elements of symmetry in a cube
- Contribution of particles at different sites in a cubic unit cells
- Practice questions

What you will learn

- Cubic lattice
- Calculation of effective number of particles in a unit cell
- Relation between atomic radius of constituent particles
- Packing efficiency
- Practice questions

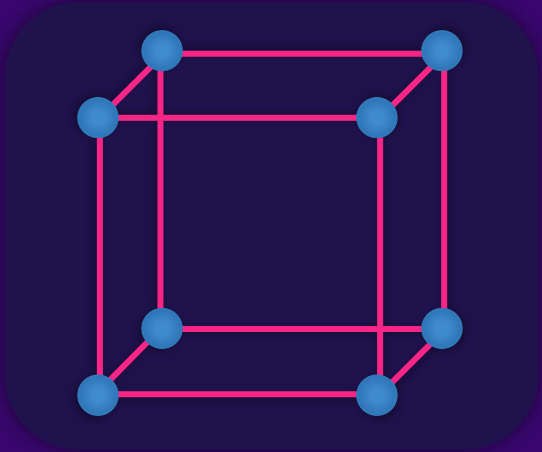
Cubic Lattice

Cubic lattice

- Simple cubic
- Body-centred cubic
- Face-centred cubic

Cubic Lattice

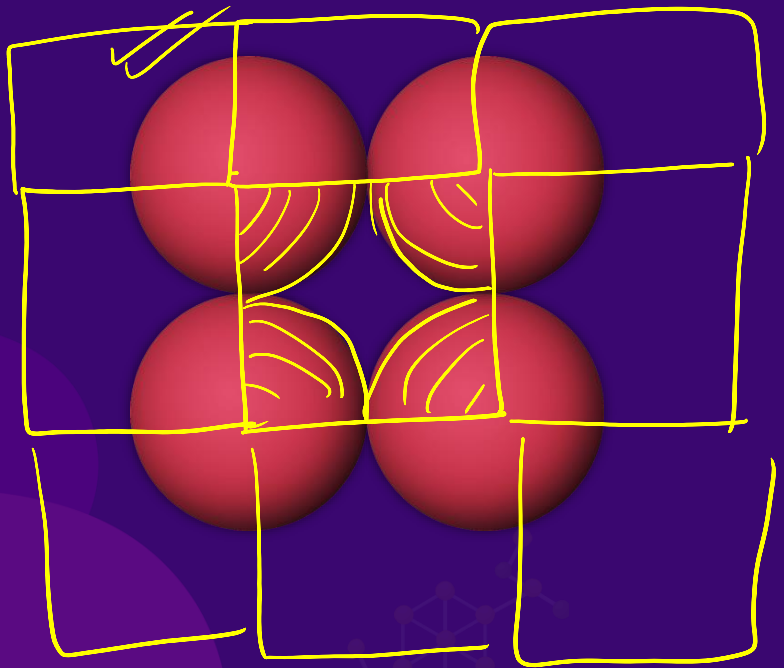
Simple/Primitive cubic unit cell



Effective number of particles in a unit cell $= 8 \times \frac{1}{8} = 1$

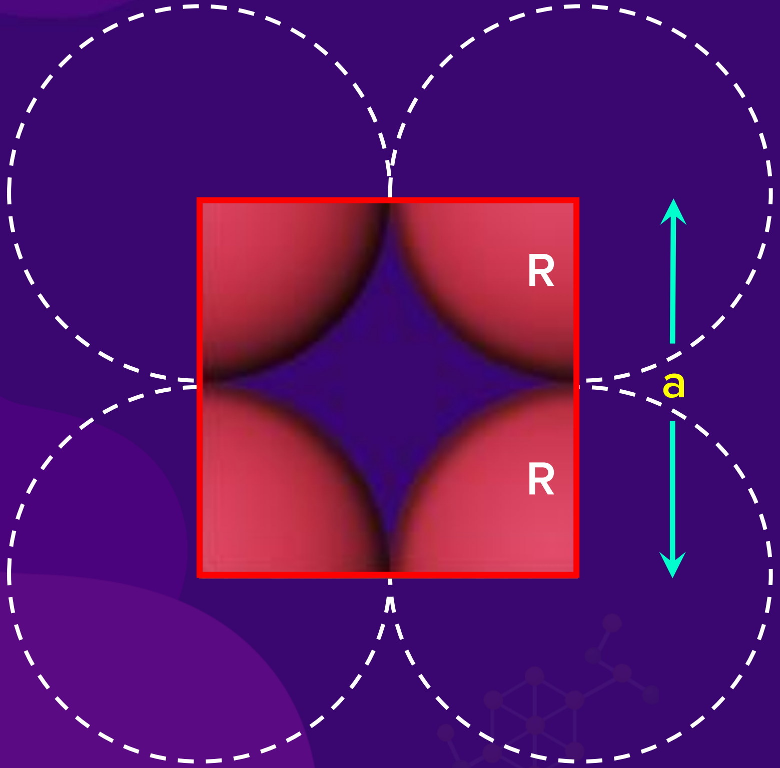
Cubic Lattice

Simple/Primitive cubic unit cell



Face of simple cubic unit cell

Cubic Lattice



$$R = \frac{a}{2}$$

Cubic Lattice

Simple/Primitive cubic unit cell

Relation between a & R

Corner atoms are touching each other

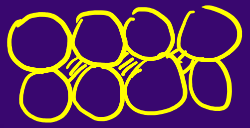
$$a = 2R$$

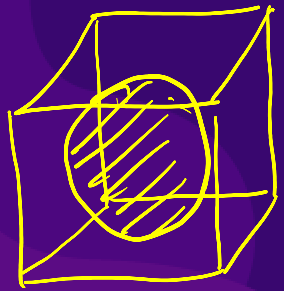
- a = Edge length of a simple cubic unit cell
- R = Radius of a particle present in that unit cell

Cubic Lattice

Packing efficiency

empty space b/w atoms/ molecules/ ions = interstitial space or voids





The percentage of the total space **filled** by the particles

Cubic Lattice

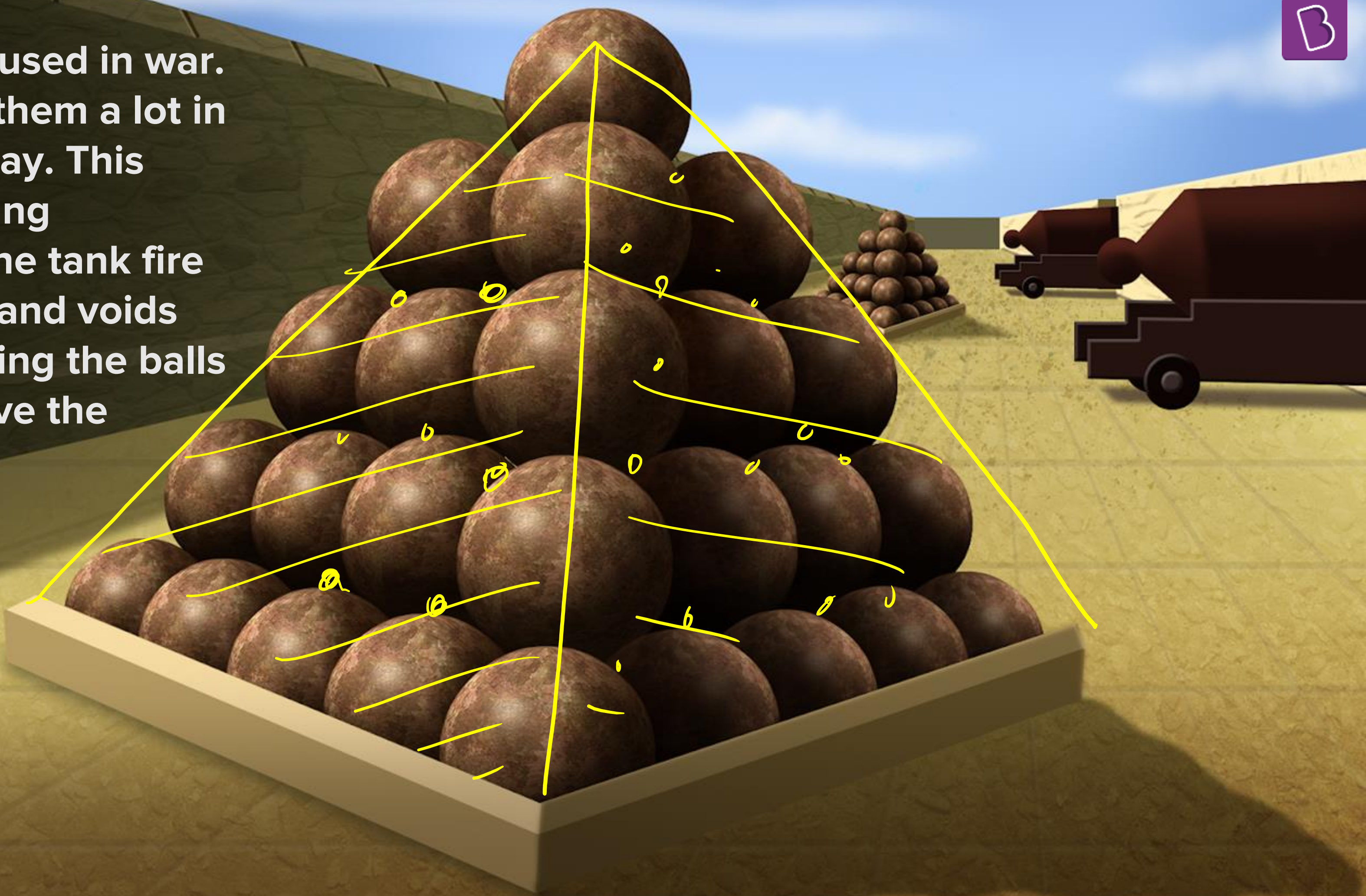
Packing efficiency

Clearly the clothes in ordered form in 2^{nd} bag has higher packing efficiency than the clothes in unordered form in the 1^{st} bag.

1^{st} bag

2^{nd} bag

Stack of balls used in war. Students see them a lot in games they play. This explains packing efficiency of the tank fire balls in a tray and voids left while placing the balls one layer above the other.



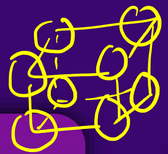
Cubic Lattice

Packing efficiency

metallic solid : Lattice point : atoms

dry ice : solid CO_2

For 3-D arrangement



molecular solid : Z = no. of molecules in a unit cell

$$P.E. = \frac{\text{Volume occupied by particles in unit cell}}{\text{Total volume of the unit cell}} \times 100$$

NaCl

Z : no. of Al_2O_3 units

$$= \frac{Z \times \text{Volume of one particle}}{\text{Total volume of the unit cell}} \times 100$$

Z = no. of particles present in one unit cell

Cubic Lattice

Packing efficiency

Total volume occupied by particles $= 1 \times \frac{4}{3}\pi R^3$

Volume of the unit cell $= a^3 = (2R)^3$

Cubic Lattice

Packing efficiency

$$P.E. = \frac{1 \times (4/3)\pi R^3}{(2R)^3} \times 100$$

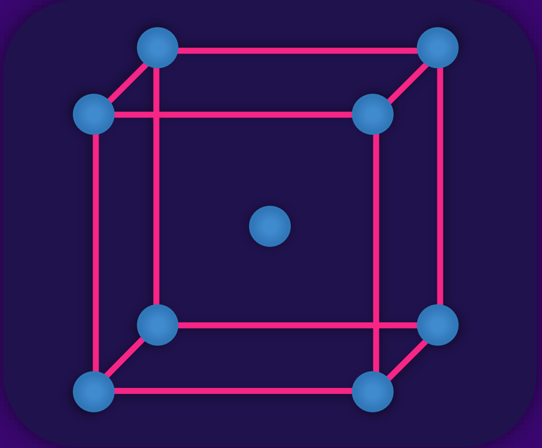
$$= \frac{\pi \times 100}{6}$$

$$\approx 52.33\%$$

η

Cubic Lattice

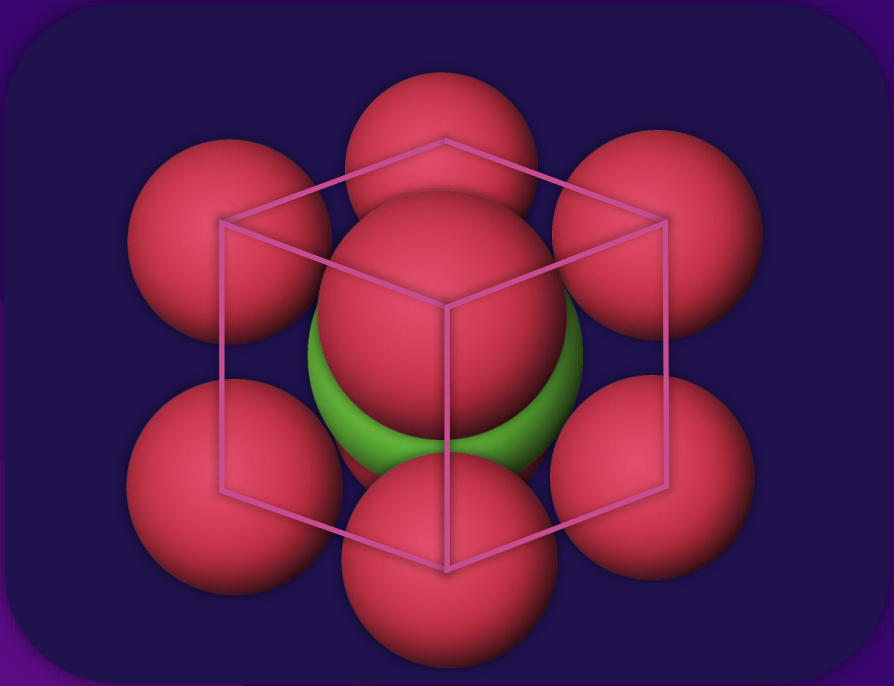
Body-centred cubic unit cell



Effective number of particles in a unit cell $= 8 \times \frac{1}{8} + (1 \times 1) = 2$

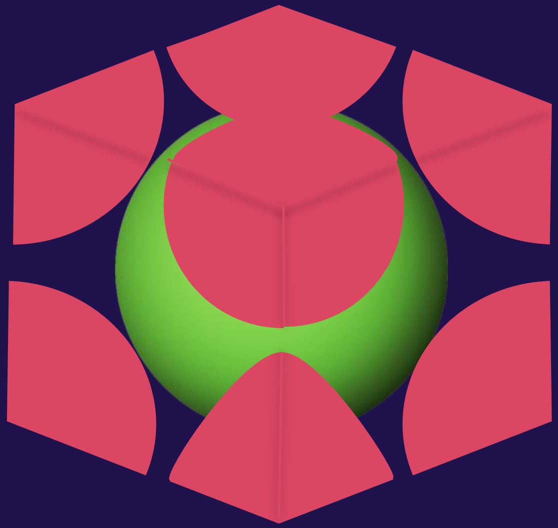
Cubic Lattice

Body-centred cubic unit cell



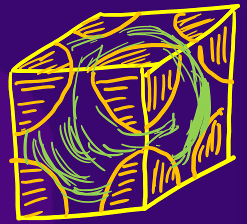
Cubic Lattice

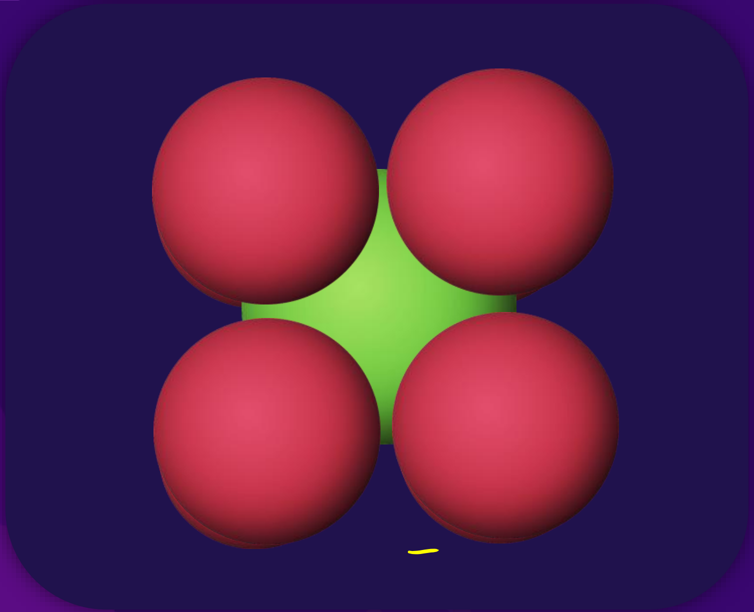
Body-centred cubic unit cell



Cubic Lattice

Body-centred cubic unit cell

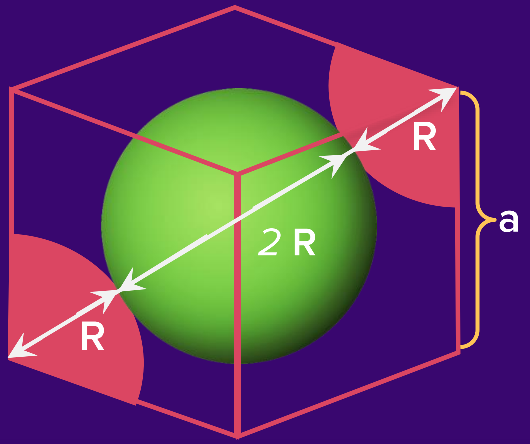


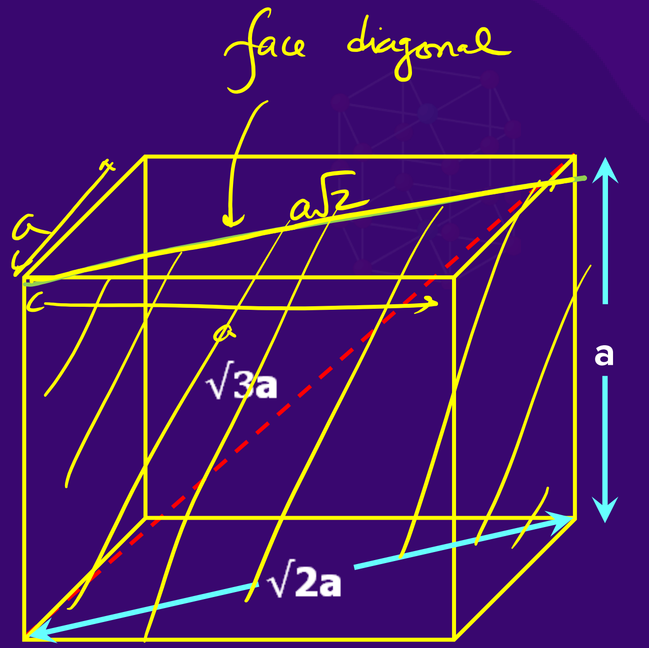


Face of body-centred cubic unit cell

Cubic Lattice

Body-centred cubic unit cell



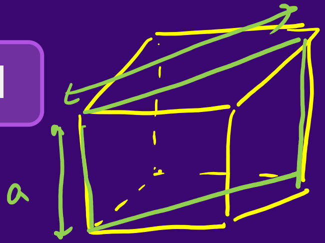


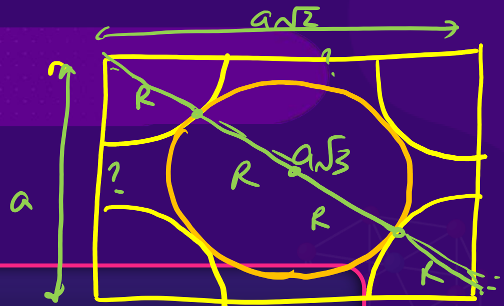
Spheres are **not touching** along **edge**.
They **touch** along the **body diagonal**.

Cubic Lattice

Body-centred cubic unit cell

Relation between a & R





Along the **body diagonal**

a√3

$\sqrt{3}a = 4R$

bcc

$a = \dfrac{4R}{\sqrt{3}}$

sc

a = 2R

Cubic Lattice

Body-centred cubic unit cell

Packing efficiency (P.E.)

For 3-D arrangement

$$\text{P.E.} = \frac{\text{Volume occupied by particles in unit cell}}{\text{Total volume of the unit cell}} \times 100$$

$$= \frac{Z \times \text{Volume of one particle}}{\text{Total volume of the unit cell}} \times 100$$

Cubic Lattice

Body-centred cubic unit cell

Packing efficiency (P.E.)

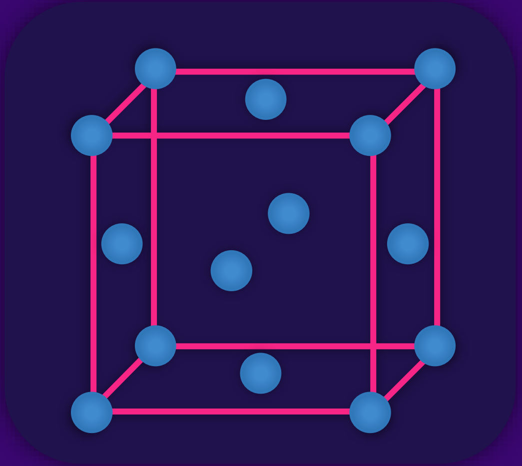
$$\text{Packing efficiency} = \frac{2 \times (4/3)\pi R^3}{(4R/\sqrt{3})^3} \times 100$$

$$= \frac{\sqrt{3}\pi \times 100}{8}$$

$$\approx 68\%$$

Cubic Lattice

Face-centred cubic unit cell

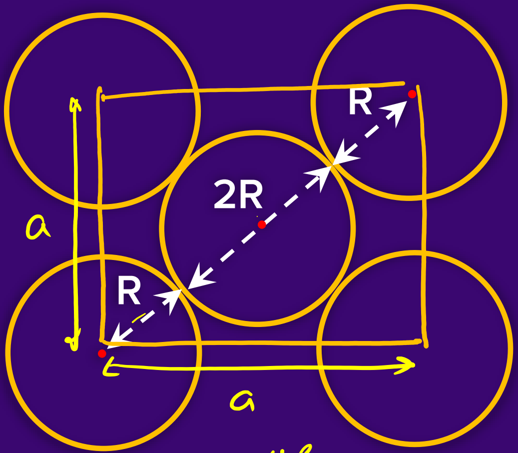


Effective number of particles in a unit cell $= 8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$

Cubic Lattice

Face-centred cubic unit cell

Relation between a and R



Face of unit cell

$a\sqrt{2} = 4R$

$a > 2R$

$\sqrt{2}a = 4R$

$\therefore$ $a = 2\sqrt{2}R$

Spheres are touching along the face diagonal

Cubic Lattice

Face-centred cubic unit cell

Packing efficiency (P.E.)

$$\text{P.E.} = \frac{4 \times (4/3)\,\pi R^3}{\left(\frac{4R}{\sqrt{2}}\right)^3} \times 100$$

$$= \frac{\pi \times 100}{3\sqrt{2}}$$

$$\approx 74\%$$

In an FCC lattice of X and Y, X atoms are present at the corners while Y atoms are at the face-centres. If one of the X atoms from a corner is replaced by Z atoms (also monovalent), then the formula of the compound would be:

a $X_7Y_{24}Z_2$

b $X_7Y_{24}Z$

c $X_{24}Y_7Z$

d $XY_{24}Z$

Solution

Corners : X : $\frac{1}{8} \times 7 = \frac{7}{8}$

fc : Y : $6 \times \frac{1}{2} = 3Y$

: Z : $\frac{1}{8}$

$X_{\frac{7}{8}} Y_3 Z_{\frac{1}{8}}$

$X_7 Y_{24} Z$

Therefore, option (b) is the correct answer.

A metal crystallises in BCC. Find the % fraction of edge length not covered and also % fraction of edge length covered by atom is:

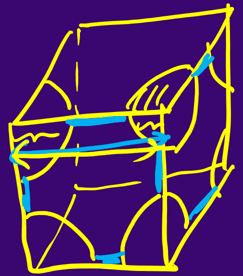
a) 10.4%

b) 13.4%

c) 86.6%

d) 89.6%

Solution



edge occupied = 2R

" length = a = $\frac{4}{\sqrt{3}}R$

% of edge length occupied = $\frac{2R}{\frac{4}{\sqrt{3}}R} \times 100$

= $\frac{\sqrt{3}}{2} \times 100\%$ = 86.6

% Edge length covered by metal atoms in a bcc lattice = 86.6%

% Edge length not covered by metal atoms in a bcc lattice = 100 – 86.6 = 13.4 %

Therefore, option b and c are the correct answers.

How many **number of atom** effectively present in a **cubic unit** formed by arrangement of **eight BCC unit cell.**

Solution

1 bcc → 2

8 " → 8 x 2 = 16

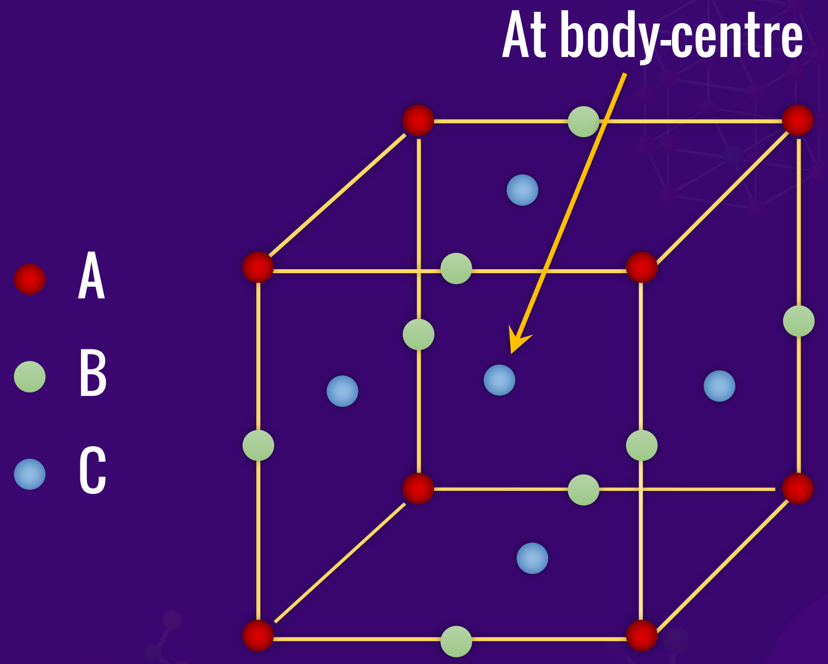
Given: The unit cell structure of compound is shown below. The **formula** of compound is:

a. $A_8B_{12}C_5$

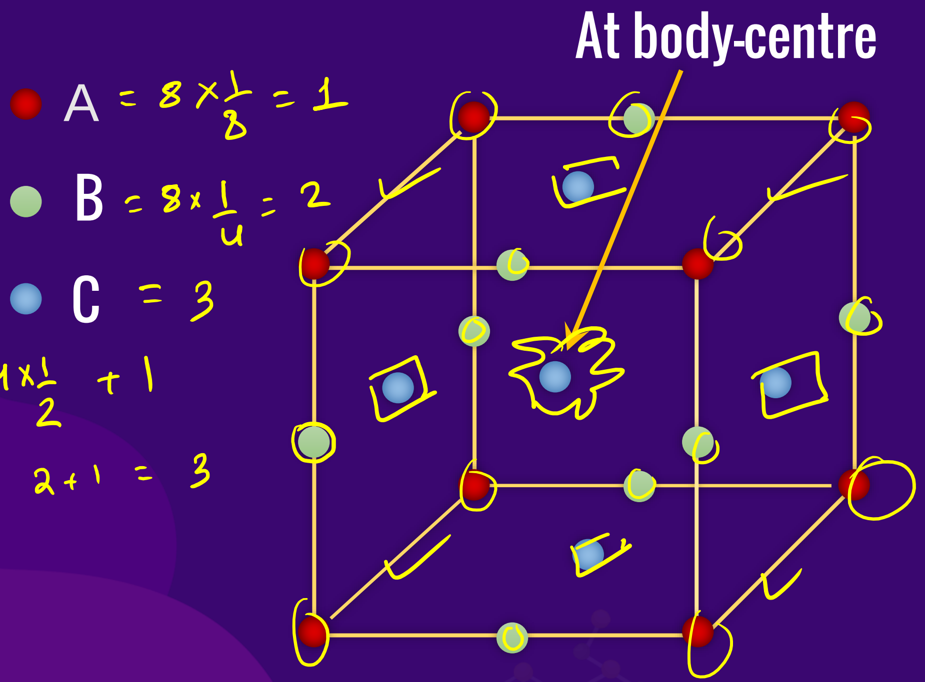
b. AB_2C_3

c. $A_2B_2C_5$

d. ABC_5



Solution

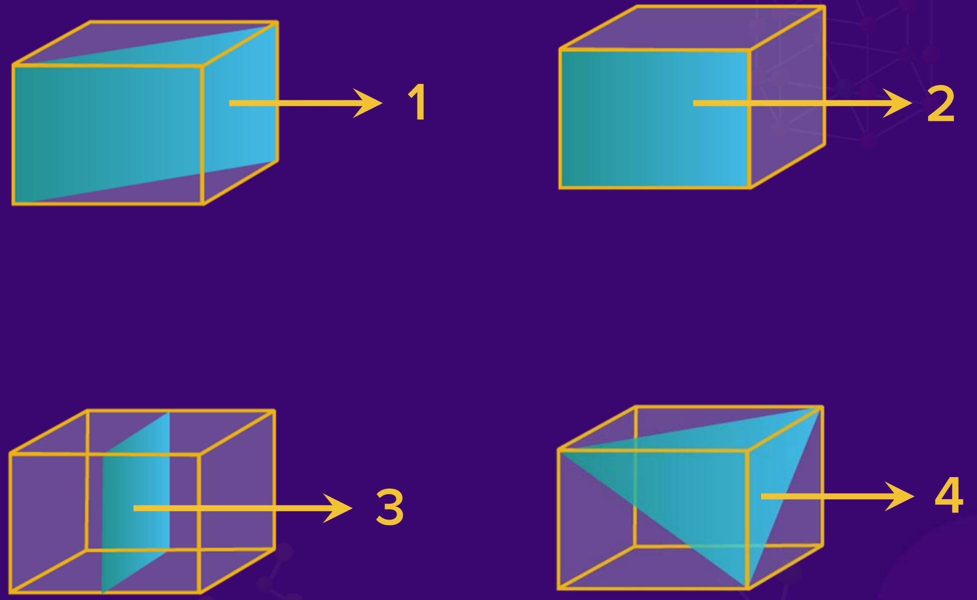


Formula of the compound = AB_2C_3

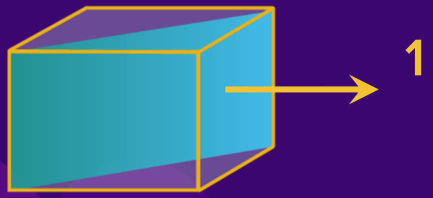
Therefore, option (b) is the correct answer.

In a BCC-arrangement, which of the marked planes have **maximum spatial density** of atoms?

- a. 1
- b. 2
- c. 3
- d. 4



Solution



In BCC $a = \frac{4}{\sqrt{3}} r$

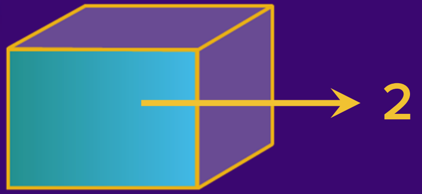
$a^2 = \frac{16}{3} r^2$

No. of circles in this plane = $\frac{1}{4} \times 4 + 1 = 2$

Area of plane 1 = $a \times \sqrt{2}a = \sqrt{2}a^2 = \sqrt{2}\left(\frac{16}{3}\right) r^2$

Area of circles in plane 1 = $2 \times \pi r^2$

$$\eta_1 = \frac{2 \times \pi r^2}{\sqrt{2}\left(\frac{16}{3}\right) r^2} \times 100 = 0.833 \times 100 = 83.3\ \%$$



In BCC $a = \frac{4}{\sqrt{3}} r$

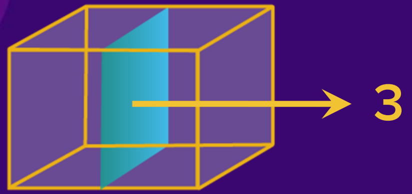
$a^2 = \frac{16}{3} r^2$

No. of circles in this plane = $\frac{1}{4} \times 4 = 1$

Area of plane 2 = $a \times a = a^2 = \frac{16}{3} r^2$

Area of circles in plane 2 = $1 \times \pi r^2$

$$\eta_2 = \frac{\pi r^2}{\left(\frac{16}{3}\right) r^2} \times 100 = 0.589 \times 100 = 58.9\ \%$$



In BCC $a = \frac{4}{\sqrt{3}} r$

$a^2 = \frac{16}{3} r^2$

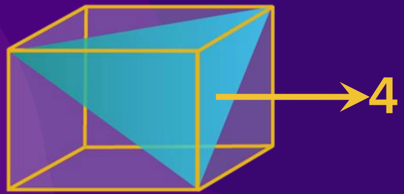
No. of circles in this plane = 1 (sphere at body center)

Area of plane 3 = $a \times a = a^2 = \frac{16}{3} r^2$

Area of circles in plane 3 = $1 \times \pi r^2$

$$\eta_3 = \frac{\pi r^2}{\left(\frac{16}{3}\right) r^2} \times 100 = 0.589 \times 100 = 58.9\ \%$$

Since the plane does not passes through the center of the cube so it cuts only three sectors at the corners of the cube which much lesser as compared to the area of whole triangular plane. So efficiency of this plane is the least among all other options.



Therefore, option (a) is the correct answer.

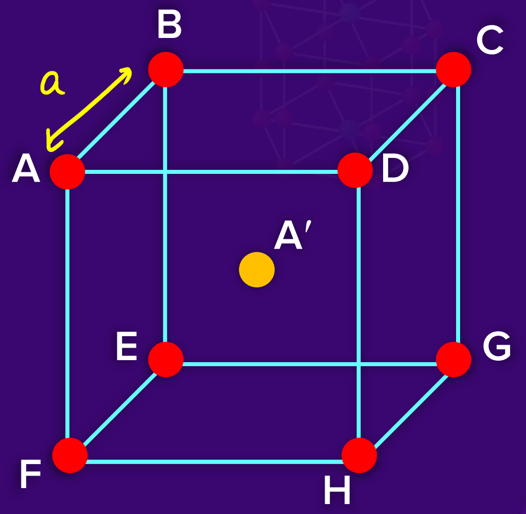
In **body-centred cubic** lattice given below, the three distance **AB, AC, and AA′** are:

a) AB = a

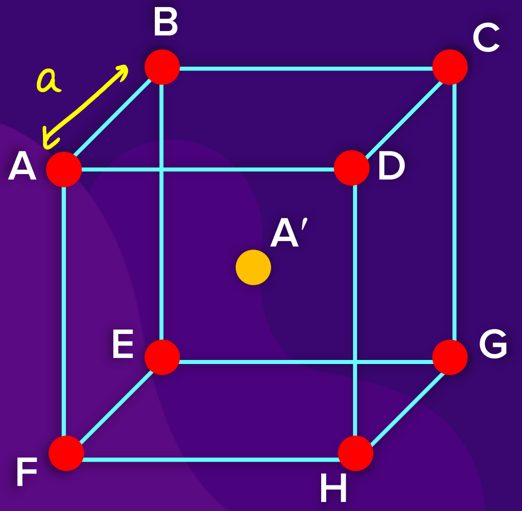
b) AC = $\sqrt{2}a$

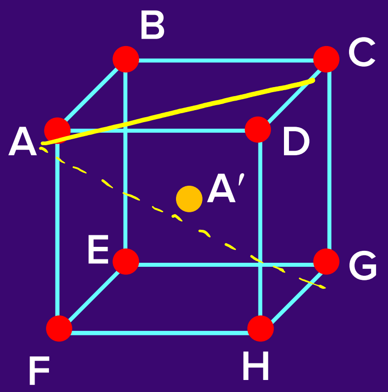
c) $AA' = \frac{\sqrt{3}a}{2}$

d) $AA' = \sqrt{3}a$



Solution





Side length of a cube AB = a

Side length of AC = √2 a

AG = Body diagonal

= $a\sqrt{3}$

$AA' = \frac{AG}{2} = \frac{a\sqrt{3}}{2}$

Therefore, options (a), (b), (c) are the correct answers.

A substance A_xB_y crystallises in a face-centred cubic lattice in which atoms 'A' occupy each corner of the cube and atoms 'B' occupy the centres of each face of the cube. Identify the correct composition of the substance A_xB_y.

a. AB_3

b. A_4B_3

c. A_3B

d. Composition can't be specified

Solution

fcc

Corner : A : $\frac{1}{8} \times 8 = 1$

fc : B

$\frac{1}{2} \times 6 = 3$

AB_3

Therefore, option (a) is the correct answer.

Keep Learning!