MATHEMATICS

NUMBER SYSTEMS

MORE ON NUMBER SYSTEM AND FACTORISATION

NOTES

What you already know

- Hierarchy of number system
- Representation of numbers on number line
- Prime numbers

What you will learn

- Prime factorisation
- Algebraic identities and its applications

Three positive integers greater than 1, have a product of 27,000 and are pairwise relative prime numbers. What is the sum of these three numbers?

Solution:

$27{,}000 = 2^3 \times 3^3 \times 5^3 \times 1$
$\Rightarrow S = 8 + 27 + 125$
$\Rightarrow S = 160$

$x, y \in \mathbf{N}$ such that $x^2 = y^2 + 2018$. What will be the number of ordered pairs (x, y) ?

Solution:

Given $x^2 = y^2 + 2018$
$\Rightarrow x^2 - y^2 = 2018$
$\Rightarrow (x - y)(x + y) = 2018$
The factors of 2018 are 1, 2, 1009, 2018.

Since 2018 is even (but it is NOT a multiple of 4), x^2 and y^2 must be both even or both odd, which in turn implies that x and y must be either both even or both odd. So there is no such value of x and y.

If the positive integers A, B, A − B, A + B are prime numbers, then what is the sum of these 4 numbers?

Solution:

A, B, A - B, A + B are prime numbers.
$\Rightarrow B = A - (A - B)$
B = 2 or B > 2
i) B > 2
$\Rightarrow$ B is odd.
A - B > 0, A > B. So, A has to be odd.
A - B, A + B are both even.
So, this case isn't possible.

ii) B = 2
A - 2, A, A + 2 are three consecutive odd primes.
They are 3, 5 and 7.
Sum = 2 + 3 + 5 + 7 = 17

Algebraic Identities

1. $(x+a)(x+b)=x^2+(a+b)x+ab$
2. $(x-a)(x-b)=x^2-(a+b)x+ab$
3. $(a+b)^2=a^2+2ab+b^2$
4. $(a-b)^2=a^2-2ab+b^2$
5. $(a+b)(a-b)=a^2-b^2$
6. $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$
7. $(a+b)^2=(a-b)^2+4ab$
8. $(a-b)^2=(a+b)^2-4ab$
9. $(a+b)^3=a^3+b^3+3ab(a+b)=a^3+b^3+3a^2b+3ab^2$
10. $(a-b)^3=a^3-b^3-3ab(a-b)=a^3-b^3-3a^2b+3ab^2$
11. $a^3+b^3=(a+b)^3-3ab(a+b)=(a+b)(a^2-ab+b^2)$
12. $a^3-b^3=(a-b)^3+3ab(a-b)=(a-b)(a^2+ab+b^2)$
13. $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$
14. $a^3+b^3+c^3-3abc=\frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$
15. If $a+b+c=0$, then $a^3+b^3+c^3=3abc$
16. $a^4-b^4=(a-b)(a^3+a^2b+ab^2+b^3)$
17. $a^5-b^5=(a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)$
18. $a^5+b^5=(a+b)(a^4-a^3b+a^2b^2-ab+b^4)$
19. $a^7-b^7=(a-b)(a^6+a^5b+a^4b^2+a^3b^3+a^2b^4+ab^5+b^6)$
20. $a^n-b^n=(a-b)\left(a^{(n-1)}+a^{(n-2)}b+a^{(n-3)}b^2+....+ab^{(n-2)}+b^{(n-1)}\right)$
21. $1+x^2+x^3+....+x^{n-1}=\frac{(x^n-1)}{x-1}$

Note

$a^n + b^n$ can be factorised only for n = odd; for n = even it cannot be factorised for any real value.

If x + y = 12 and xy = 35, what is ($x^4 + y^4$)?

Solution:

$$x^4+y^4=(x^2+y^2)^2-2x^2y^2$$

$$=\left((x+y)^2-2xy\right)^2-2(xy)^2=(12^2-2\cdot35)^2-2\cdot35^2=5476-2450=3026$$

$x^5 - 1 = 0$, find its factors.

Solution:

$$x^5 - 1 = 0$$
$$= (x - 1)(x^4 + x^3 + x^2 + x + 1)$$

If x + 1/x = 2, find the value of $x^{10} + x^{21}$, x > 0

Solution:

$$= x + \frac{1}{x} = 2 \text{ (Given)}$$

Multiply both sides by x

$$\Rightarrow x^2 + 1 = 2x$$
$$\Rightarrow x^2 - 2x + 1 = 0$$
$$\Rightarrow (x - 1)^2 = 0, \text{ therefore } x = 1$$
$$\text{So, } x^{10} + x^{21} = 2$$

Concept Check :

(1) Determine whether 67 is a prime number.
(2) $a^2 + b^2 + c^2 = 2(a - b + 2c) - 6$, find $(a + b + c)$.

Summary sheet

Key results

1. $1 + x + x^2 + x^3 + ... + x^{n-1} = \frac{(x^n - 1)}{x - 1}$;(Sum of n terms of Geomtric Progression Series)

Key takeaways

1. $a^n - b^n$ is always a multiple of $(a - b)$.
2. $a^n + b^n$ can be factorised only for n = odd number.
3. $a^n - b^n = (a - b)(a^{(n-1)} + a^{(n-2)}b + a^{(n-3)}b^2 + ... + ab^{(n-2)} + b^{(n-1)})$
4. If $(x - a)^2 + (y - b)^2 + (z - c)^2 = 0$, then $x = a, y = b, z = c$.

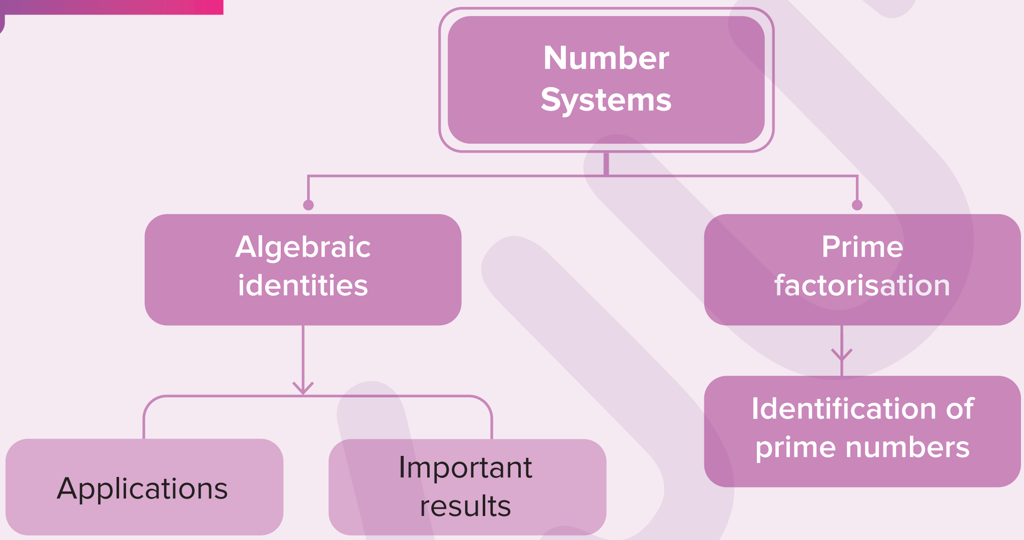
Self-assessment

1. Expand $\left(x^{17}-1\right)$.

2. If $x+\frac{4}{x}=4$, find the value of x^3.

3. If $x+\frac{1}{x}=\sqrt{3}$, find the value of x^6+1.

Mind map



Answers

Concept Check

1.

Step 1: Square root of 67 is approx 8.

Step 2: Check divisibility of 67 by prime numbers 2, 3, 5, and 7..

Step 3: Since 67 is not a multiple of any of the above prime numbers, it is a prime number.

2.
$$a^2+b^2+c^2=2(a-b+2c)-6$$
$$\Rightarrow \left(a^2-2a+1\right)+\left(b^2+2b+1\right)+\left(c^2-4c+4\right)=0$$
$$\Rightarrow (a-1)^2+(b+1)^2+(c-2)^2=0$$
$$\Rightarrow a=1,\ b=(-1),\ c=2$$
$$\Rightarrow a+b+c=2$$

Self-assessment

1. $x^{17} - 1^{17}$

$= (x-1)(x^{16} + x^{15} + x^{14} \ldots x + 1)$

2. $x + \frac{4}{x} = 4$, where x is a non-zero real number

$x^2 + 4 = 4x$

$(x-2)^2 = 0$

$x = 2$, therefore $x^3 = 8$

3. $x + \frac{1}{x} = \sqrt{3}$

Applying cube on both sides,

$\Rightarrow \left(x + \frac{1}{x}\right)^3 = 3\sqrt{3}$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 3\sqrt{3}$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\sqrt{3} = 3\sqrt{3}$

$\Rightarrow \frac{x^6 + 1}{x^3} = 0$

$\Rightarrow x^6 + 1 = 0$