BYJU'S Classes

Relations and Functions

Solving problems on Periodic functions

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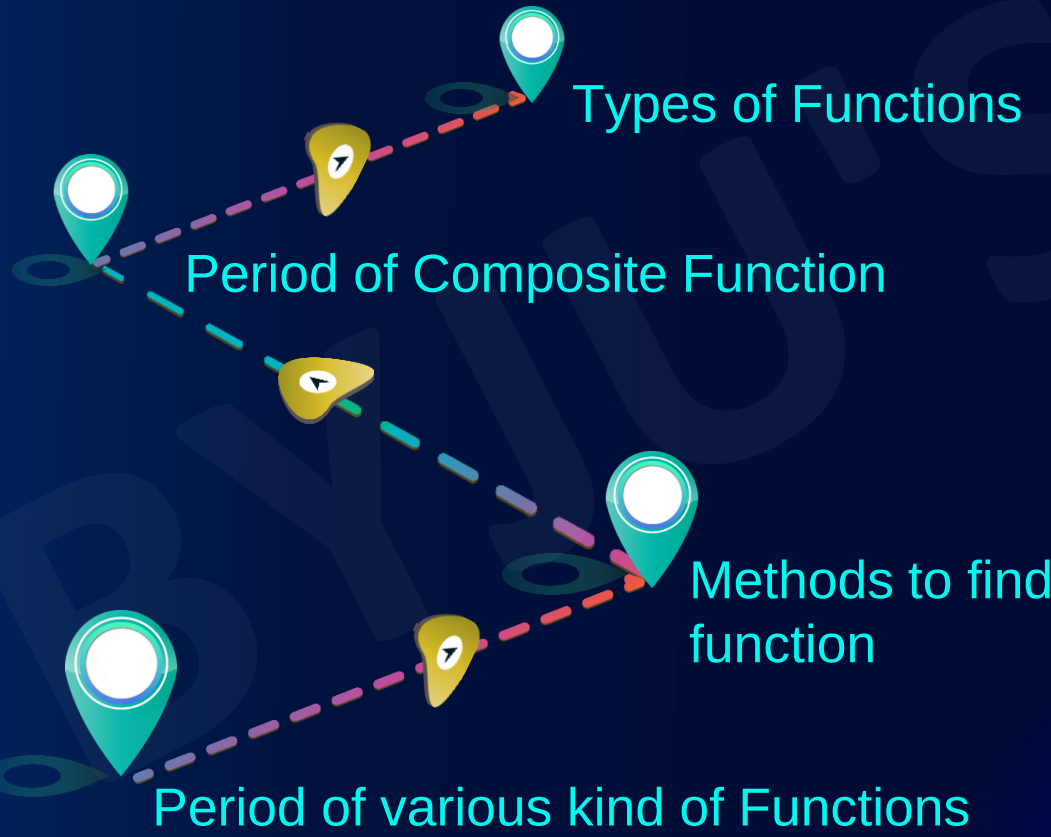
Road Map



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Recall

Functions	Nature of n	Fundamental period T
$\sin^n x\,, \cos^n x\,,$ $\operatorname{cosec}^n x, \sec^n x$	$n =$ odd	2π
	$n =$ even	π
$\tan^n x\,, \cot^n x$	$n =$ even/odd	π
$\lvert \sin^n x \rvert, \lvert \cos^n x \rvert,$ $\lvert\operatorname{cosec}^n x\rvert, \lvert\sec^n x\rvert,$ $\lvert\tan^n x \rvert, \lvert \cot^n x \rvert$	$n =$ even/odd	π

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Functions	T
$\lvert\sin x\rvert + \lvert\cos x\rvert, \lvert\tan x\rvert + \lvert\cot x\rvert,$ $\lvert\sec x\rvert + \lvert\operatorname{cosec} x\rvert$	$\frac{\pi}{2}$
$\sin^E x + \cos^E x \ (E = \text{even} > 2)$	$\frac{\pi}{2}$
$x - [x] = \{x\}$	1
Constant Function	Fundamental period is undefined
Dirichlet Function $f(x)$ $\begin{cases} 1, x = Q \ \text{(Rational number)} \\ -1, x = Q' \ \text{(irrational number)} \end{cases}$	Periodic but no fundamental Period

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When constant is involved to $f(x)$

- If a function $f(x)$ has a period T, then

$f(x) \pm k$
$f(x \pm k)$
$kf(x)$
$k^{f(x)}$
$\frac{1}{f(x)}$
$\sqrt{f(x)}$
$\frac{f(x)}{k}$

} Period remains T

Example : If $f(x) = sinx$

$f(x) + 2 = sinx + 2$

$\frac{f(x+2)}{3} = \frac{\sin(x+2)}{3}$

$\frac{1}{f(x)} = \frac{1}{sinx}$

$2f(x) = 2sinx$

$2^{f(x)} = 2^{sinx}$

$\sqrt{f(x)} = \sqrt{sinx}$

In all these examples period remain 2π

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Illustration : Find the period of following functions.

- $f(x) = \sec^3 x$

  Degree of $secant$ is odd so period is $T = 2\pi$

- $f(x) = \sec^3(3x)$

  Degree of $secant$ is odd and coefficient of x is 3 so period is $T = \frac{2\pi}{3}$

- $f(x) = \sec^3\left(\frac{x}{4}\right)$

  Degree of $secant$ is odd and coefficient of x is $\frac{1}{4}$ so period is $T = \frac{2\pi}{\frac{1}{4}} = 8\pi$

- $f(x) = \tan\left(\frac{x}{n!}\right)$

  Degree of tangent is odd and coefficient of x is $\frac{1}{n!}$ so period is $T = \frac{\pi}{\frac{1}{n!}} = n!\,\pi$

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Illustration : Find the period of following functions.

- $f(x) = \frac{\{3x-4\}}{5} + 3$

While adding, subtracting or dividing by any non-zero constant, period does no change. But, here coefficient of x is 3 so period is $T = \frac{1}{3}$

- $f(x) = \frac{\cos^2(\pi x-3)}{5}$

Degree of $cosine$ is even and coefficient of x is π so period is $T = \frac{\pi}{\pi} = 1$

- $f(x) = \sqrt{\sin\left(\frac{x}{2} - 1\right)}$

Here square root does not affect the period.

Power of $sine$ is odd and coefficient of x is $\frac{1}{2}$ so period is $T = \frac{2\pi}{\frac{1}{2}} = 4\pi$

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Illustration : Find the period of following functions.

- $f(x) = \cos\left(\frac{x-\pi}{3}\right) = cos\left(\frac{x}{3} - \frac{\pi}{3}\right)$

  Here, $-\frac{\pi}{3}$ is the angle that does not affect the period of function.

  Degree of $cosine$ is odd and coefficient of x is $\frac{1}{3}$ so period is $T = \frac{2\pi}{\frac{1}{3}} = 6\pi$

- $f(x) = \frac{|tan(8x+9)|}{7} + \frac{\pi}{2}$

  Period of $|tanx|$ is π and coefficient of x is 8 so period is $T = \frac{\pi}{8}$

- $f(x) = \frac{7}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}} = \frac{7}{\sqrt{\sin x}}$

  Here, square root does not affect the period.

  Degree of $sine$ is odd so period is $T = 2\pi$

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When 2 or more functions are Added/Subtracted/multiplied

- When 2 or more functions are given,

  $h(x) = f(x) \pm g(x)$ or $f(x) \,.\, g(x)$ or combination of these ,

  Where T_1, T_2 be the period of $f(x)$ and $g(x)$ respectively.

  Period of $h(x)$ can be calculated by taking L.C.M of T_1 & T_2

  $T =$ L.C.M. (T_1, T_2) (Period T may or may not be fundamental period)

Note

L.C.M is only possible when T_1 & T_2 are of same type

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Illustration : If $f(x) = \sin\frac{x}{2} + \cos\frac{x}{2^2}$ then find period of $f(x)$.

Solution:

$$f(x) = \sin\frac{x}{2} + \cos\frac{x}{2^2}$$

$$\Rightarrow f(x) = \sin\frac{x}{2} + \cos\frac{x}{4}$$

Period of $\sin\frac{x}{2}$ is $T_1 = \frac{2\pi}{\frac{1}{2}} = 4\pi$

Period of $\cos\frac{x}{4}$ is $T_2 = \frac{2\pi}{\frac{1}{4}} = 8\pi$

Period of $f(x)$ is given by $T = \text{L.C.M.}\,(4\pi, 8\pi) = 8\pi$

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Illustration : $f(x) = 2^{\tan \pi[x] + \sin \pi x + \cos^2 \frac{\pi x}{4}}$ then find Period of $f(x)$.

([.] denote the greatest integer function)

Solution:

$$f(x) = 2^{\tan \pi[x] + \sin \pi x + \cos^2 \frac{\pi x}{4}}$$

$[x]$ always gives integer value so we can replace it by integer n

$$\tan \pi[x] = \tan n\pi = 0$$

Period of $\sin \pi x \rightarrow T_1 = \frac{2\pi}{\pi} = 2$

Period of $\cos^2 \frac{\pi x}{4} \rightarrow T_2 = \frac{\pi}{\frac{\pi}{4}} = 4$

Period of $f(x)$ is given by

T = L.C.M. $(2,4) = 4$

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Illustration : If $f(x), g(x)$ are two periodic functions having period 7 and 11 respectively, then find the period of $h(x) = f(x)g\left(\frac{x}{5}\right) - g(x)f\left(\frac{x}{5}\right)$.

Solution: $h(x) = f(x)g\left(\frac{x}{5}\right) - g(x)f\left(\frac{x}{5}\right)$

Period of $f(x)$ is given by $T_1 = 7$

Period of $g\left(\frac{x}{5}\right)$ is given by $T_2 = \frac{11}{\frac{1}{5}} = 55$

Period of $g(x)$ is given by $T_3 = 11$

Period of $f\left(\frac{x}{5}\right)$ is given by $T_4 = \frac{7}{\frac{1}{5}} = 35$

Period of $h(x)$ is given by

$\therefore T = \text{L.C.M}\,(T_1, T_2, T_3, T_4)$

$\therefore T = \text{L.C.M}\,(7, 55, 11, 35)$

$\Rightarrow T = 7 \times 5 \times 11 = 385$

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Illustration : If $h(x) = \sin\frac{x}{3} + \text{cosec}^2\left(\frac{2x}{5}\right)$ then find period of $h(x)$.

Solution:

$$h(x) = \sin\frac{x}{3} + \text{cosec}^2\left(\frac{2x}{5}\right)$$

Period of $sin\frac{x}{3} \rightarrow T_1 = \frac{2\pi}{\frac{1}{3}} = 6\pi$

Period of $cosec^2\left(\frac{2x}{5}\right) \rightarrow T_2 = \frac{\pi}{\frac{2}{5}} = \frac{5\pi}{2}$

Period of $h(x)$ is given by

$$T = \text{L.C.M.}\left(6\pi, \frac{5\pi}{2}\right) = \frac{\text{L.C.M.}\ (6\pi, 5\pi)}{\text{H.C.F.}\ (1, 2)} \quad \left(\because \text{L.C.M.}\left(\frac{a}{b}, \frac{c}{d}\right) = \frac{\text{L.C.M.}\ (a, c)}{\text{H.C.F.}\ (b, d)}\right)$$

$$T = \frac{30\pi}{1} \Longrightarrow \quad T = 30\pi$$

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Illustration : If $f(x) = \sin^2 3x + \cos^2 4x$ then find period of $f(x)$.

Solution:

$$f(x) = \sin^2 3x + \cos^2 4x$$

Period of $sin^2\, 3x$ is $T_1 = \frac{\pi}{3}$

Period of $cos^2\, 4x$ is $T_2 = \frac{\pi}{4}$

Period of $f(x)$ is given by

$$T = \text{L.C.M.}\left(\frac{\pi}{3}, \frac{\pi}{4}\right) = \frac{\text{L.C.M.}\ (\pi, \pi)}{\text{H.C.F.}\ (3, 4)} \left(\because \text{L.C.M.}\left(\frac{a}{b}, \frac{c}{d}\right) = \frac{\text{L.C.M.}\ (a,c)}{\text{H.C.F.}\ (b,d)}\right)$$

$$= \frac{\pi}{1}$$

$$\Rightarrow T = \pi$$

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Illustration :Find period of the function $h(x) = \sec x + \{x\}$.

Solution:

$h(x) = \sec x + \{x\}$

Period of $sec\ x$ is 2π

Period of $\{x\}$ is 1

Period of $h(x)$ is given by

$T =$ L.C.M. $(2\pi, 1)$

2π is irrational number and 1 is rational number. So, the L.C.M does not exist.

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Illustration : If $h(x) = \sec x + x$ then find period of $h(x)$.

Solution:

$h(x) = \sec x + x$

$\sec x$ is a periodic function and period is 2π

x is non periodic function

$h(x)$ is sum of $sec\ x$ (periodic) and x (non periodic)

Periodic + Non Periodic = Non-Periodic

So, $h(x)$ is non periodic function

$\Rightarrow$Period of the function is not defined.

Note

Sum of Periodic and non Periodic function is always non periodic

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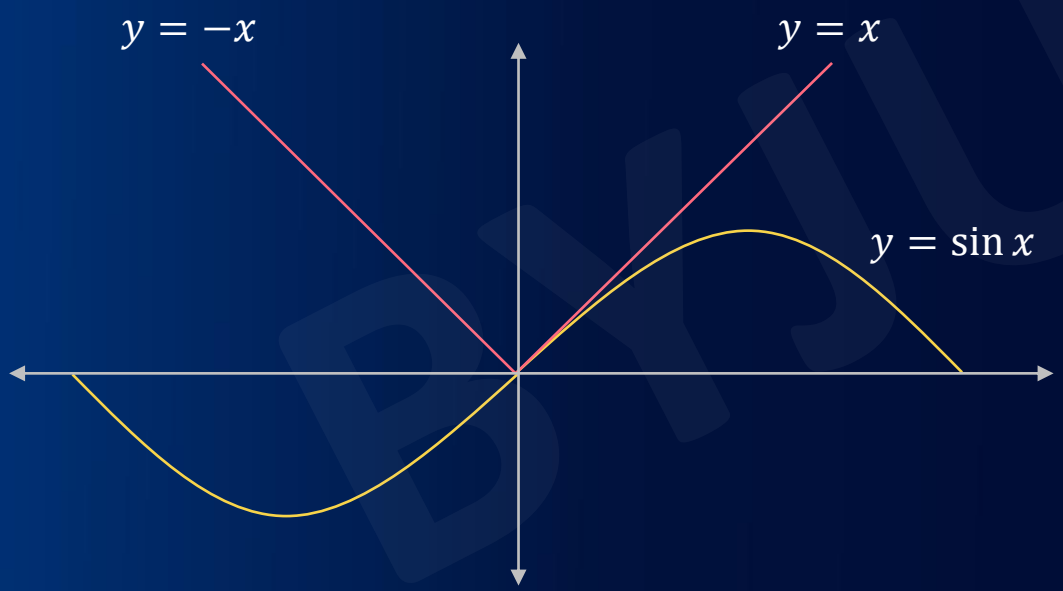
Illustration : If $f(x) = \max(\sin x, |x|)$ then find period of $f(x)$.

Solution: $f(x) = \max(\sin x, |x|) = |x|$



From graph we can see that

$max(sin\, x, |x|) = |x|$

So, $f(x) = |x|$

$|x|$ is non periodic function

$\Rightarrow f(x)$ is Non Periodic

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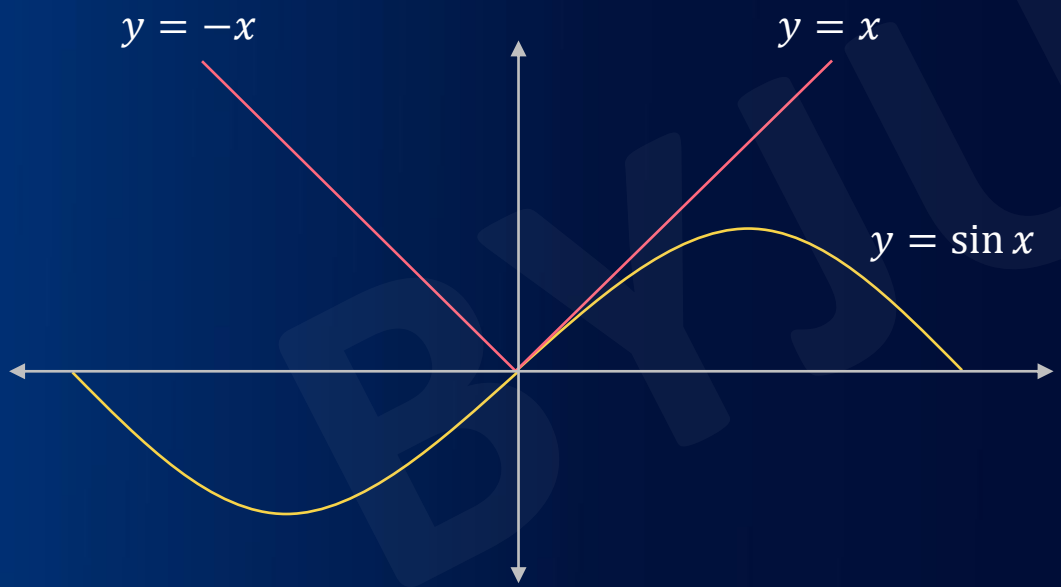
Illustration : If $f(x) = \min(\sin x, |x|)$ then find period of $f(x)$.

Solution: $f(x) = \min(\sin x, |x|) = \sin x$



From graph you can see that

$\min(\sin x, |x|) = \sin x$

So, $f(x) = sin\, x$

Period of $\sin x$ is 2π

$T = 2\pi$

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Illustration : Find period of $f(x) = \frac{1}{1-\cos x}$

Solution:

$$f(x) = \frac{1}{1-\cos x}$$

$$= \frac{1}{2\sin^2\frac{x}{2}}$$

Here degree of $\sin x$ is even and coefficient of x is $\frac{1}{2}$

So Period of $\sin^2\frac{x}{2}$ is $= \frac{\pi}{\frac{1}{2}} = 2\pi$

Period of $f(x) = \frac{\pi}{\frac{1}{2}} = 2\pi$

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Illustration : Find period of $f(x) = \frac{\sin 12x}{1+\cos^2 6x}$

Solution:

$$f(x) = \frac{\sin 12x}{1+\frac{1}{2}+\frac{1}{2}\cos 12x} \quad \left(\therefore \cos^2 6x = \frac{1+\cos 12x}{2}\right)$$

$$= \frac{\sin 12x}{\frac{3}{2}+\frac{1}{2}\cos 12x}$$

Period of $\sin 12x = \frac{2\pi}{12} = \frac{\pi}{6}$

Period of $\cos 12x = \frac{2\pi}{12} = \frac{\pi}{6}$

Period of $f(x) =$ L.C.M. $\left(\frac{\pi}{6}, \frac{\pi}{6}\right)$ So, period $= \frac{\pi}{6}$

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Illustration : Find period of $f(x) = \sec^2 x + \text{cosec}^3 x$

Solution:

$$f(x) = \sec^2 x + \text{cosec}^3 x$$

$$= \frac{1}{\cos^2 x} + \frac{1}{\sin^3 x}$$

Here degree of $\cos x$ is even. So, Period of $\cos^2 x$ is π

Degree of $\sin x$ is odd. So, Period of $\sin^3 x$ is 2π

Period of $f(x) =$ L.C.M.$(\pi, 2\pi)$

Period $= 2\pi$

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Illustration : Find period of $f(x) = 1 - \frac{\sin^2 x}{1+\cot x} - \frac{\cos^2 x}{1+\tan x}$

Solution:

$$f(x) = 1 - \frac{\sin^3 x}{\sin x + \cos x} - \frac{\cos^3 x}{\cos x + \sin x}$$

$$= 1 - \frac{(\sin^3 x + \cos^3 x)}{\sin x + \cos x}$$

$$= 1 - \frac{(\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x \cos x)}{(\sin x + \cos x)}$$

$$= 1 - 1 + \sin x \cos x$$

$$= \frac{1}{2}\sin 2x$$

Degree of $\sin 2x$ is odd and coefficient of x is 2 so Period of $f(x)$ is π

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Illustration : Find period of $f(x) = \sin 2x + \cos 3x + \tan 5x$

Solution:

$f(x) = \sin 2x + \cos 3x + \tan 5x$

Period of $\sin 2x = \frac{2\pi}{2} = \pi$

Period of $\cos 3x = \frac{2\pi}{3}$

Period of $\tan 5x = \frac{\pi}{5}$

Period of $f(x)$ is given by

$$T = \text{L.C.M.}\left(\frac{2\pi}{2}, \frac{2\pi}{3}, \frac{\pi}{5}\right) = \frac{\text{L.C.M.}\ (2\pi, 2\pi, \pi)}{\text{H.C.F.}\ (2, 3, 5)} \qquad T = 2\pi$$

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Illustration : Find period of $f(x) = \sin \pi x + \{x\}$.

Solution :

$f(x) = \sin \pi x + \{x\}$

Period of $\sin \pi x = \frac{2\pi}{\pi} = 2$

Period of $\{x\} = 1$

Period of $f(x)$ is given by

T = L.C.M. $(2, 1)$

$T = 2$

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Illustration : Find period of $f(x) = \sin 3x + \{x\}$.

Solution :

Period of $\sin 3x = \frac{2\pi}{3}$

Period of $\{x\} = 1$

Period of $f(x)$ is given by

$T = \text{L.C.M.} \left(\frac{2\pi}{3}, 1\right)$

Also, L.C.M of an irrational number and a rational number is not defined.

$\Rightarrow f(x) = \sin 3x + \{x\}$ is not periodic

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Illustration : Find period of $f(x) = \sin^2\left(\frac{x}{4}\right) + \cos^2\left(\frac{x}{3}\right)$.

Solution :

Here degree of sine is even and coefficient of x is $\frac{1}{4}$

So, Period of $\sin^2\left(\frac{x}{4}\right) = \frac{\pi}{\frac{1}{4}} = 4\pi$

Here degree of cosine is even and coefficient of x is $\frac{1}{3}$

So, Period of $\cos^2\left(\frac{x}{3}\right) = \frac{\pi}{\frac{1}{3}} = 3\pi$

Period of $f(x)$ is given by

$T = \text{L.C.M.}\ (4\pi, 3\pi) \Rightarrow T = 12\pi$

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Illustration : Find the period of $f(x) = 2^{(5\pi\{x\})} + \tan \pi[x]$, where [.] represents G.I.F

Solution:

$f(x) = 2^{(5\pi\{x\})} + \tan \pi[x]$

$\tan \pi[x] = \tan n\pi = 0, n \in \mathbb{Z}, [x] = n$

Period of $\{x\} = 1$

$\Rightarrow$ Period of $f(x)$ is 1

So, $T = 1$

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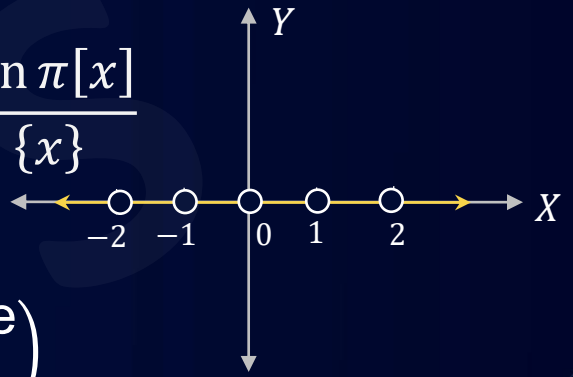
Illustration : Find the period of $f(x) = \frac{\sin \pi[x]}{\{x\}}$

Solution: $f(x) = \frac{\sin \pi[x]}{\{x\}}$



Here $\{x\} \in (0,1), x \neq \mathbb{I}$

$\sin \pi[x] = \sin n\pi = 0$ $(\because [x]$ always gives integer value $[x] = n, n \in \mathbb{I})$

$\Rightarrow f(x) = 0$

Domain of $\{x\}$ is $x \in \mathbb{R} - \mathbb{I}$. $(\because \{x\} = 0$ for $x = \mathbb{I})$

Domain of $f(x) \in \mathbb{R} - \mathbb{I}$.

So, period of $f(x)$ is 1.

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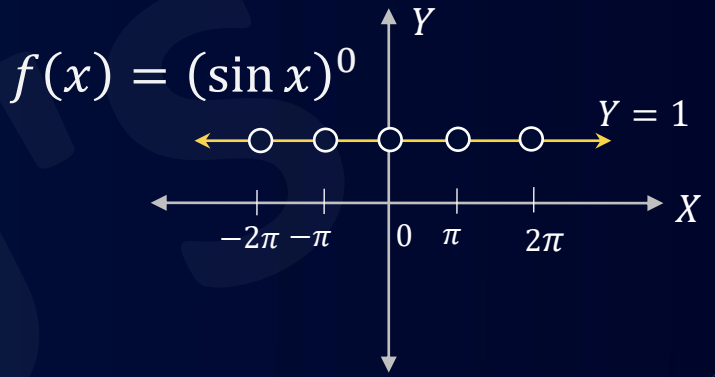
Illustration : Find the period of $f(x) = (\sin x)^0$



Solution:

$f(x) = (\sin x)^0$

Here Domain of $f(x)$ is $x \in \mathbb{R} - \{n\pi\}$.

For $x = n\pi$, $f(x)$ is of the form 0^0 (indeterminant form)

$\Rightarrow f(x) = 1 \ \forall \ x \in \mathbb{R} - \{n\pi\}$.

So, period of $f(x)$ is π.

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Illustration : $f(x) = \left[x + \frac{1}{2}\right] + \left[x - \frac{1}{2}\right] + 2[-x]$, then find period of $f(x)$.

Solution: $f(x) = \left[x + \frac{1}{2}\right] + \left[x - \frac{1}{2}\right] + 2[-x]$

$$\Rightarrow f(x) = \left(x + \frac{1}{2}\right) - \left\{x + \frac{1}{2}\right\} + \left(x - \frac{1}{2}\right) - \left\{x - \frac{1}{2}\right\} + 2(-x - \{-x\})$$

$$(\because x = [x] + \{x\} \Rightarrow [x] = x - \{x\})$$

$$\Rightarrow f(x) = 2x - \left\{x + \frac{1}{2}\right\} - \left\{x - \frac{1}{2}\right\} - 2x - 2\{-x\}$$

$$\Rightarrow f(x) = -\underbrace{\left\{x + \frac{1}{2}\right\}}_{T_1 = 1} - \underbrace{\left\{x - \frac{1}{2}\right\}}_{T_2 = 1} - \underbrace{2\{-x\}}_{T_3 = \left|\frac{1}{-1}\right| = 1}$$

$$\Rightarrow T = \text{L.C.M.}\ (1, 1, 1) = 1$$

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Illustration : Find period of $f(x) = [x] + \left[x + \frac{1}{3}\right] + \left[x + \frac{2}{3}\right] - 3[x] + 10$

Solution:

We know, $[x] + \left[x + \frac{1}{n}\right] + \left[x + \frac{2}{n}\right] + \cdots + \left[x + \frac{n-1}{n}\right] = [nx]$

$$\Rightarrow [x] + \left[x + \frac{1}{3}\right] + \left[x + \frac{2}{3}\right] = [3x]$$

$$f(x) = [3x] - 3[x] + 10$$

$$= 3x - \{3x\} - 3x + 3\{x\} + 10$$

$$= -\{3x\} + 3\{x\} + 10$$

$T_1 = \frac{1}{3}$ $T_2 = 1$

$$\Rightarrow T = \text{L.C.M.}\left(\frac{1}{3}, 1\right) = 1$$

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When period of the trigonometric functions can not be determined directly

Then, apply following method :

- Step I : $f\left(\frac{\pi}{2}+x\right)=f(x)$, If satisfies then period of $f(x)$ is $T=\frac{\pi}{2}$.
- Step II : $f(\pi+x)=f(x)$, If satisfies then period of $f(x)$ is $T=\pi$.
- If step I and step II does not satisfy then period of $f(x)$ is $T=2\pi$.

Note : This method is applicable only for trigonometric functions.

If Step I dose not satisfy then only check step II

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Illustration : Find period of $f(x) = \cos(\cos x) + \cos(\sin x)$

Solution:

Now, By using verification method,

$$f\left(\frac{\pi}{2} + x\right) = \cos\left(\cos\left(\frac{\pi}{2} + x\right)\right) + \cos\left(\sin\left(\frac{\pi}{2} + x\right)\right)$$

$$= \cos(-\sin x) + \cos(\cos x)$$

$$= \cos(\sin x) + \cos(\cos x)$$

$$= f(x)$$

So, period is $\frac{\pi}{2}$.

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Illustration : Find period of $f(x) = \frac{|\sin x| + |\cos x|}{|\sin x - \cos x|}$

Solution:

Now, By using verification method,

$$f\left(\frac{\pi}{2} + x\right) = \frac{|\cos x| + |-\sin x|}{|\cos x + \sin x|} = \frac{|\cos x| + |\sin x|}{|\cos x + \sin x|} \neq f(x)$$

$$f(\pi + x) = \frac{|-\sin x| + |-\cos x|}{|-\sin x + \cos x|}$$

$$= \frac{|\sin x| + |\cos x|}{|-(\sin x - \cos x)|} = \frac{|\sin x| + |\cos x|}{|\sin x - \cos x|}$$

$$= f(x)$$

So, period of $f(x)$ is π.

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Illustration : If $f(x) = \frac{\sin nx}{\sin\left(\frac{x}{n}\right)}$ is periodic and period is 4π, then find n.

Solution: $f(x) = \frac{\sin nx}{\sin\left(\frac{x}{n}\right)}$

Period of $\sin nx$ is $\frac{2\pi}{n}$.

Period of $\sin\left(\frac{x}{n}\right)$ is $\frac{2\pi}{\frac{1}{n}} = 2n\pi$.

Period of $f(x) = \text{L.C.M.}\left(\frac{2\pi}{n}, 2n\pi\right)$

$$T = \text{L.C.M.}\left(\frac{2\pi}{n}, 2n\pi\right) = \frac{\text{L.C.M.}\ (2\pi,\ 2n\pi)}{\text{H.C.F.}\ (n,1)}$$

$$= 2n\pi$$

$$\Rightarrow 2n\pi = 4\pi \qquad \Rightarrow n = \pm 2$$

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Period of a composite function

$f(g(x))$

Both f and g are non periodic	f is periodic and g are non periodic	f is non periodic and g is periodic	Both f and g are periodic
N.P(N.P)	P(N.P)	N.P(P)	P(P)
↓	↓	↓	↓
Non periodic	Non periodic	Periodic [Period of inside function i.e., $g(x)$]	Periodic

Note : N.P means non periodic

P means periodic

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Illustration : Find period of $f(x) = [\sin 3x] + [\cos 6x]$

Solution: $f(x) = [\sin 3x] + [\cos 6x]$

For $[\sin 3x]$:

$g(x) = [.]$ (greatest integer function) is non periodic function

$h(x) = \sin 3x$ is periodic function

$g(h(x)) = [\sin 3x] =$ N.P(P)

For $[\cos 6x]$:

$g(x) = [.]$ (greatest integer function) is non periodic function

$h(x) = \cos 6x$ is periodic function

$g(h(x)) = [\cos 6x] =$ N.P(P)

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Period of $[\sin 3x] = \frac{2\pi}{3}$

Period of $[\cos 6x] = \frac{2\pi}{6}$

Period of $f(x) = \text{L.C.M.} \left(\frac{2\pi}{3}, \frac{2\pi}{6}\right)$

$$= \frac{\text{L.C.M.}\ (2\pi, 2\pi)}{\text{H.C.F.}\ (3,6)} \Longrightarrow \text{Period of } f(x) = \frac{2\pi}{3}$$

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Illustration : Find period of $f(x) = \sin(\cos x)$

Solution: $f(x) = \sin(\cos x)$

For $\sin(\cos x)$:

$g(x) = \sin x$ is periodic function

$h(x) = \cos x$ is periodic function

$g(h(x)) = \sin(\cos x) =$ P(P), so $\sin(\cos x)$ is periodic

Now, by using verification method

$$f\left(\frac{\pi}{2} + x\right) = \sin(-\sin x) = -\sin(\sin x) \neq f(x)$$

$$f(\pi + x) = \sin(-\cos x) = -\sin(\cos x) \neq f(x)$$

$$f(2\pi + x) = \sin(\cos x) = \sin(\cos x) = f(x)$$

So, period of $f(x)$ is 2π.

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Illustration : Find period of $f(x) = \sin\sqrt{x}$

Solution :

$f(x) = \sin\sqrt{x}$

↓

P(N.P) gives N.P

For $\sin\sqrt{x}$:

$g(x) = \sin x$ is periodic function

$h(x) = \sqrt{x}$ is non periodic function

$f(x) = g\big(h(x)\big) = \sin\sqrt{x} =$ P(N.P)

So, $\sin\sqrt{x}$ is non periodic function

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Illustration : Find period of $f(x) = \cos x^2$.

Solution :

$f(x) = \cos x^2$

↓

P(N.P) gives N.P

For $\cos x^2$:

$g(x) = cosx$ is periodic function

$h(x) = x^2$ is non periodic function

$f(x) = g(h(x)) = \cos x^2 =$ P(N.P)

So, $\cos x^2$ is non periodic function

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Illustration : Find period of $f(x) = \log(\cos 3x)$

Solution :

$$f(x) = \log(\cos 3x)$$

$\downarrow$

N.P(P) gives P

For $\log(\cos 3x)$:

$g(x) = logx$ is non periodic function

$h(x) = \cos 3x$ is periodic function

$f(x) = g\big(h(x)\big) = \log(\cos 3x) =$ N.P(P)

So, period of $f(x) =$ period of $h(x)$

$$T = \frac{2\pi}{3}$$

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Illustration : Find period of $f(x) = \sin\{x\}$

Solution : $f(x) = \sin\{x\}$

P(P) gives P

For $\sin\{x\}$:

$g(x) = sinx$ is periodic function

$h(x) = \{x\}$ is periodic function

$f(x) = g\big(h(x)\big) = \sin\{x\} =$ P(P)

Here $\{x\}$ is not trigonometric function

So, Period of $f(x)$ = period of $\{x\}$

$$T = 1$$

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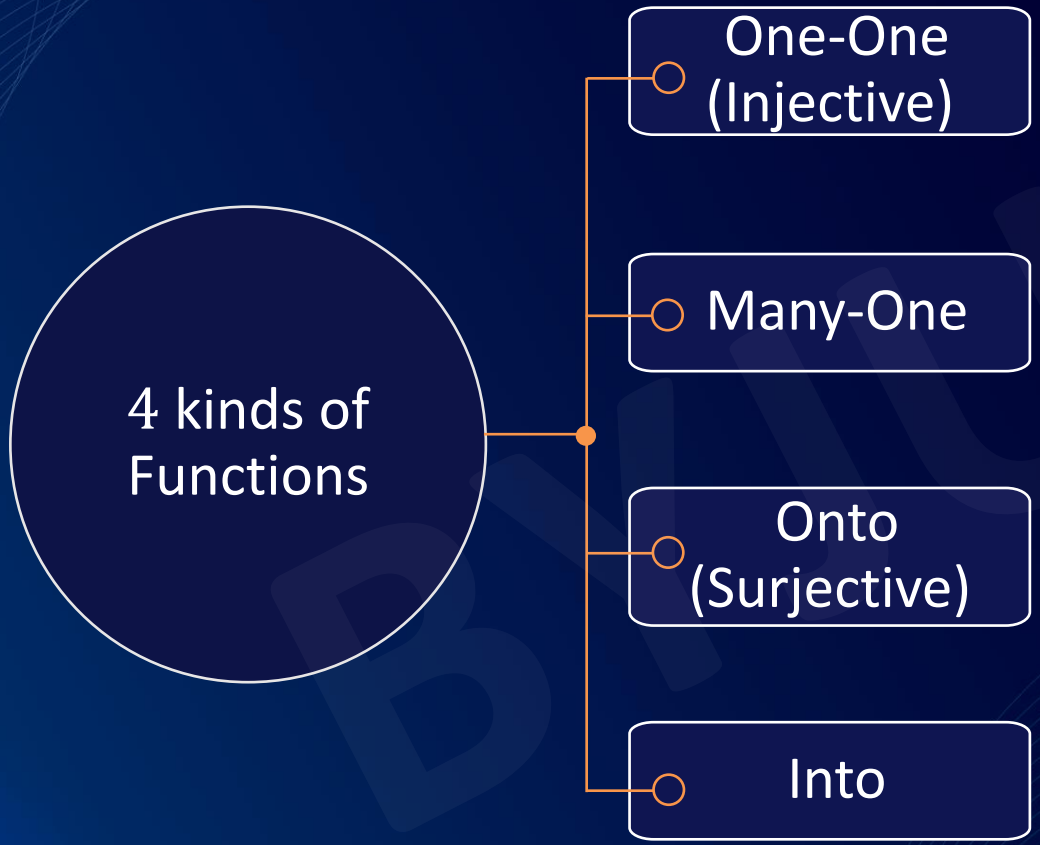

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Notes :

A function can be of one of these four types :

- One-one, onto (injective and surjective) also called as Bijective functions.
- One – one, into (injective but not surjective)
- Many – one, onto (surjective but not injective)
- Many – one, into (neither surjective nor injective)

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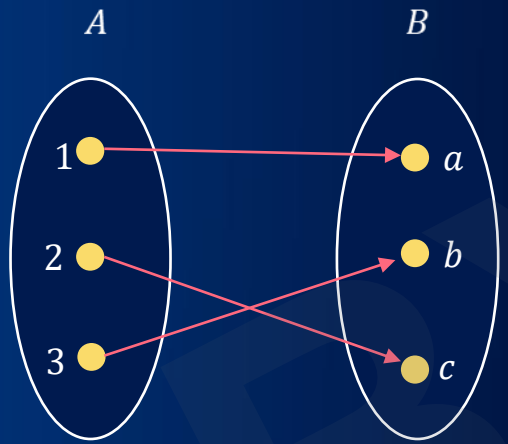
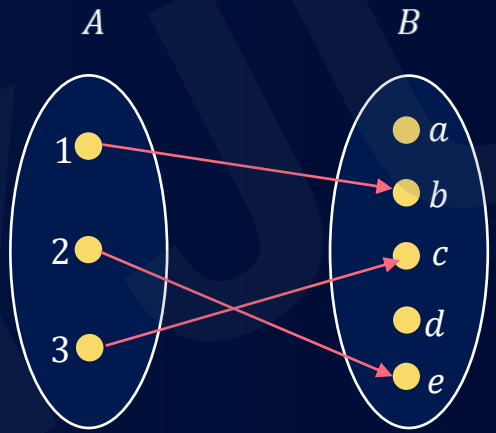
One – one function (Injective function/ Injective mapping) :

A function f: $A \rightarrow B$ is said to be a one-one function if different elements of set A have different f images in set B





Here you can see that different elements of set A have different images in B

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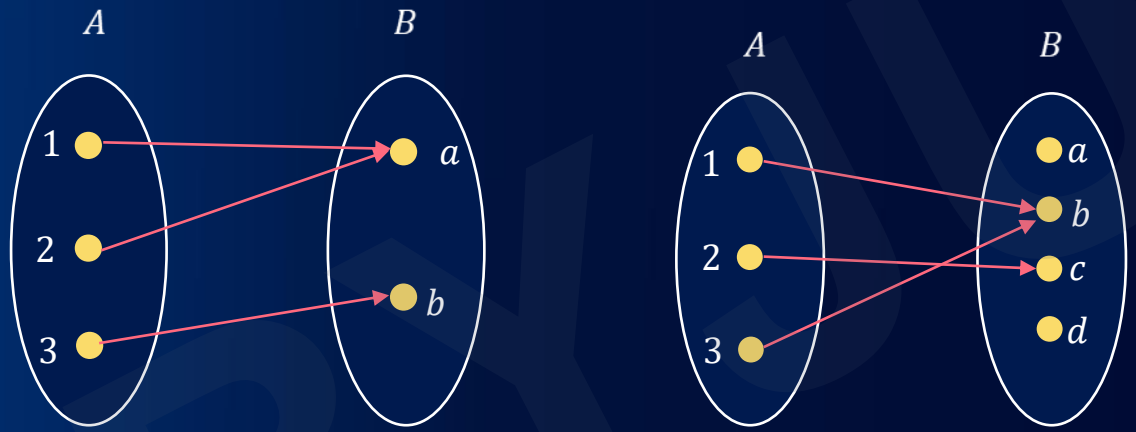
Many – one function :

A function f: $A \rightarrow B$ is said to be a many-one function if there exist at least two or more elements of set A that have same f images in set B



In 1st figure , elements 1 & 2 of set A have same image in set B so the function is many-one.

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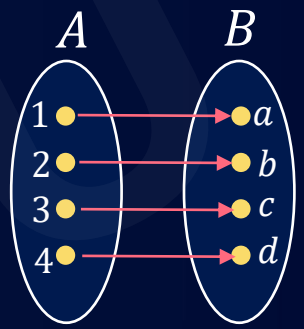
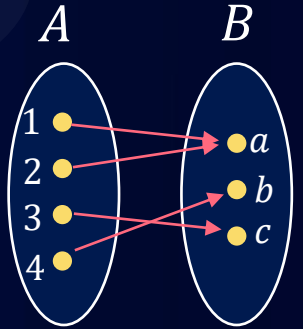
Onto function (surjective mapping):

If the function $f: A \to B$ is such that each element in B (co-domain) has at least one pre-image in A, then we say that f is an'onto' function.

- Or, if range of f = Co – domain of f.
- $f: A \to B$ is surjective iff $\forall\, b \in B$, there exists some $a \in A$ such that $f(a) = b$.



One to one, Onto



Many to one, Onto

- If co-domain is not given, then co-domain of function can be taken as R.

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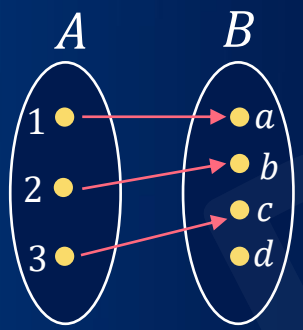
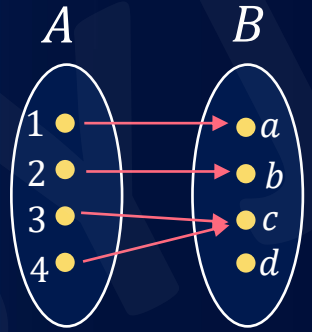
Into function:

If the function $f: A \to B$ is such that there exists at least one element in B (co-domain) which is not the image of any element in domain (A), then f is an 'into' function.

- Or, if range of f $\neq$ Co – domain of f.



One to one, Into



Many to one, Into

If a function is onto, then it can not be into and vice – versa.
If function is one- one, then it can not be many-one and vice-versa

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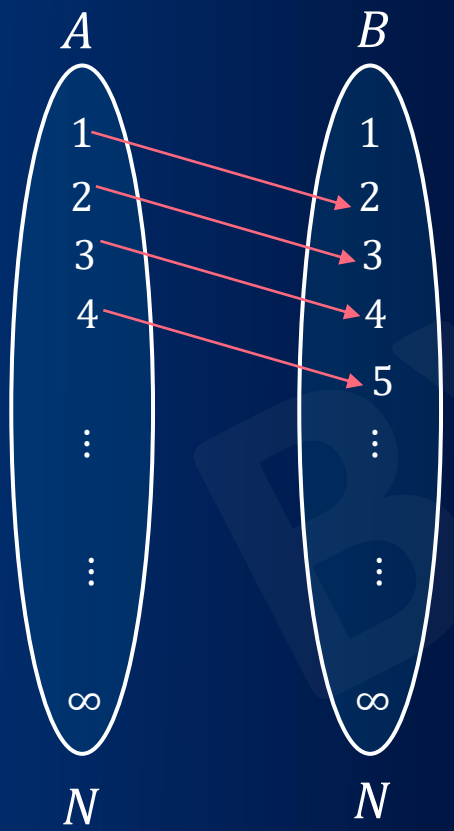
Illustration : Identify the nature of the function $f: N \to N$ such that $f(x) = x + 1$.

Solution: $f(x) = x + 1$



Here, element 1 in set B has no pre-image in set A

So,range of f $\neq$ Co – domain of f $\Rightarrow$ f is Into

Different elements of set A have different images in set B

So, function is one-one

Hence, function is One-one, Into

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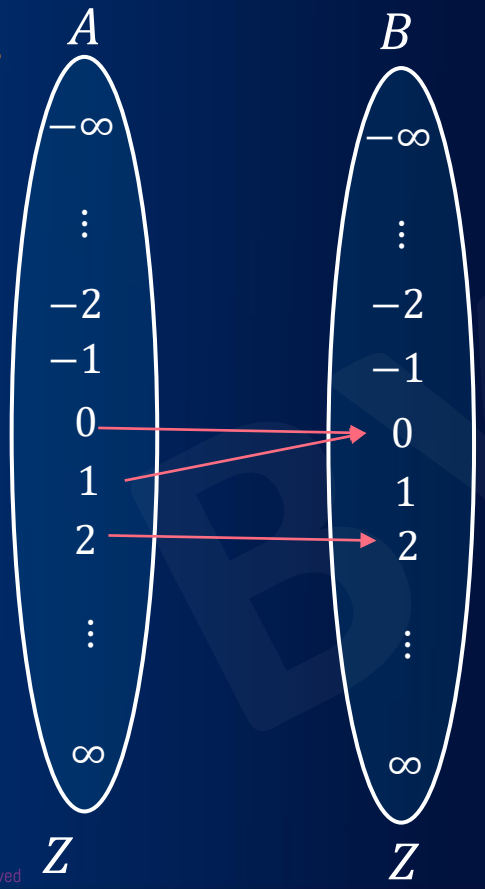
Illustration : Identify the nature of the function $f: Z \to Z$ such that $f(x) = x^2 - x$.

Solution:



$f(0) = 0 \qquad f(1) = 0 \qquad f(2) = 2$

For $f(x) = 1 \Rightarrow x^2 - x = 1$

$\Rightarrow x^2 - x - 1 = 0$

$\Rightarrow x^2 = \frac{1 \pm \sqrt{1+4}}{2}$

$\Rightarrow x = \frac{1 \pm \sqrt{5}}{2} \notin z$

So, element 1 in set B has no pre-image in set A

range of $f \neq$ Co – domain of $f \Rightarrow f$ is Into

Elements 0 & 1 of set A have same image in set B

So, function is many-one

Hence, Function is Many-one, Into

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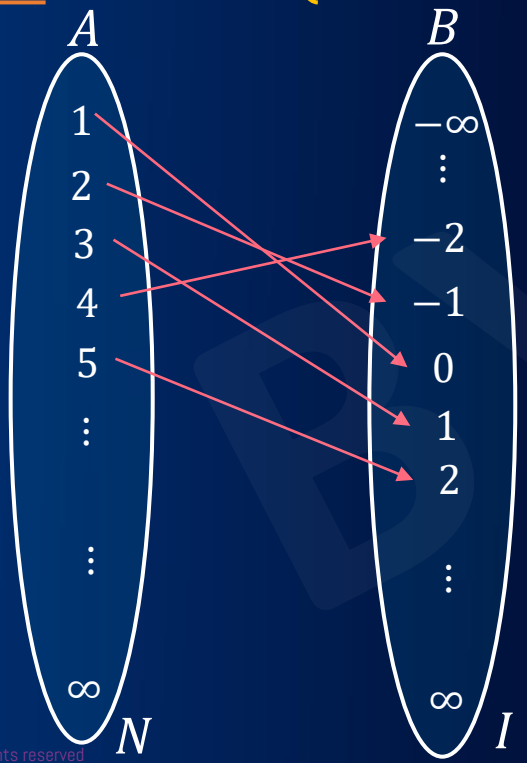
Illustration : Identify the nature of the function $f: N \to I$ such that

$$f(x) = \begin{cases} \frac{n-1}{2} & n = \text{odd} \\ \frac{-n}{2} & n = \text{even} \end{cases}$$

Solution:



$n = 1$	$f(x) = 0$
$n = 2$	$f(x) = -1$
$n = 3$	$f(x) = 1$
$n = 4$	$f(x) = -2$
$n = 5$	$f(x) = 2$

So, range of f = Co – domain of f $\Rightarrow$ f is Onto

Different elements of set A have different image in set B so function is one-one.

Function is One-one, Onto

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Summary Sheet

- When 2 or more functions are given like $h(x) = f(x) \pm g(x)$ or $h(x) = f(x) \,.\, g(x)$ or combination of these ,then period of $h(x)$ is $T =$ L.C.M. (T_1, T_2), where T_1 is the period of f(x) and T_2 is the period of $g(x)$. T need not be fundamental period.
- LCM(T_1, T_2) is only possible when T_1 and T_2 are of same type
- When period of the trigonometric functions can not be determined directly Then, apply following method :
  - Step I : $f\left(\frac{\pi}{2} + x\right) = f(x)$, If satisfies then period of $f(x)$ is $T = \frac{\pi}{2}$.
  - Step II : $f(\pi + x) = f(x)$, If satisfies then period of $f(x)$ is $T = \pi$.
  - If step I and step II does not satisfy then period of $f(x)$ is $T = 2\pi$.

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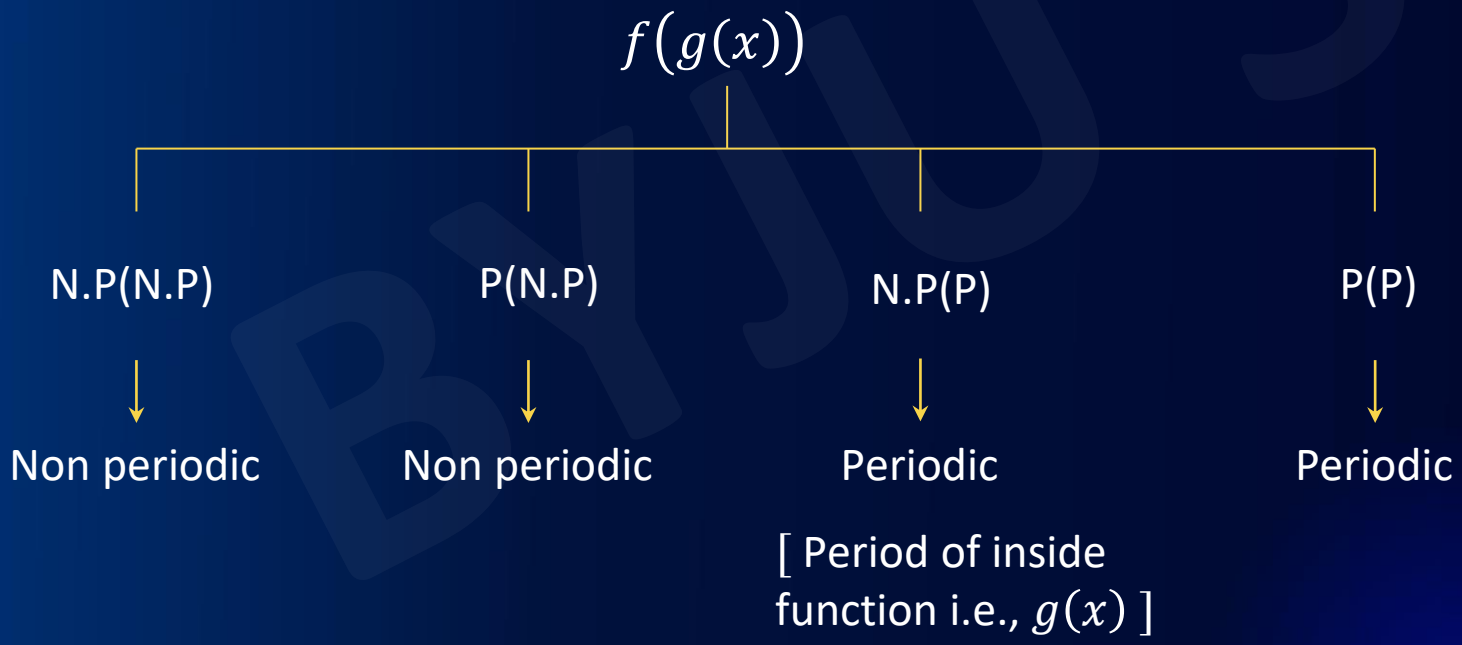
Summary Sheet

For composite function (fog(x) type)



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Summary Sheet

One – one function (Injective function/ Injective mapping) :

A function f: $A \rightarrow B$ is said to be a one-one function if different elements of set A have different f images in set B

Many – one function :

A function f: $A \rightarrow B$ is said to be a many-one function if there exist at least two or more elements of set A that have same f images in set B

Onto function (surjective mapping):

If range of f = Co – domain of f then $f(x)$ is said to be Onto function .

Into function:

If range of f ≠ Co – domain of f then $f(x)$ is said to be Into function .

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