WELCOME TO

BYJU'S CLASSES

ELECTROSTATICS

S16: Electric flux

What you already know

- Electric field
- Electric potential
- Equipotential surface
- Electric dipole
- Electric field lines

What you will learn

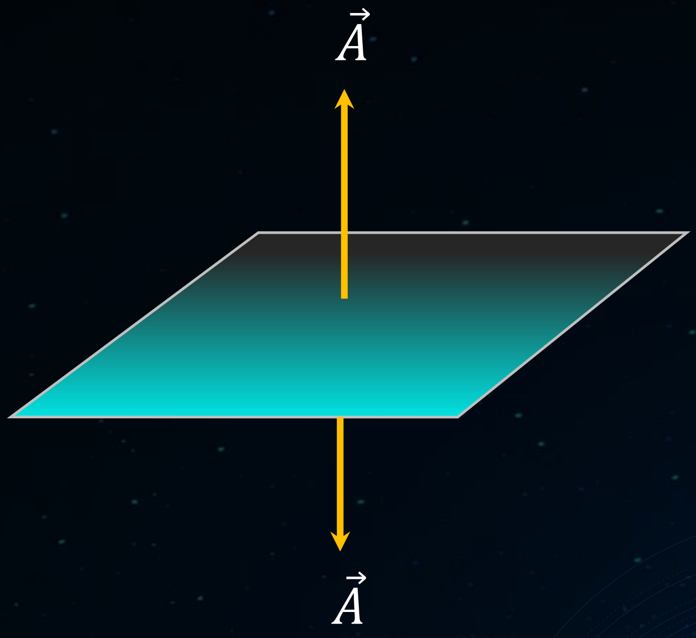
- Area vector
  - -Open surface
  - -Closed surface
- Electric flux
  - -Curved surface
  - -Closed surface

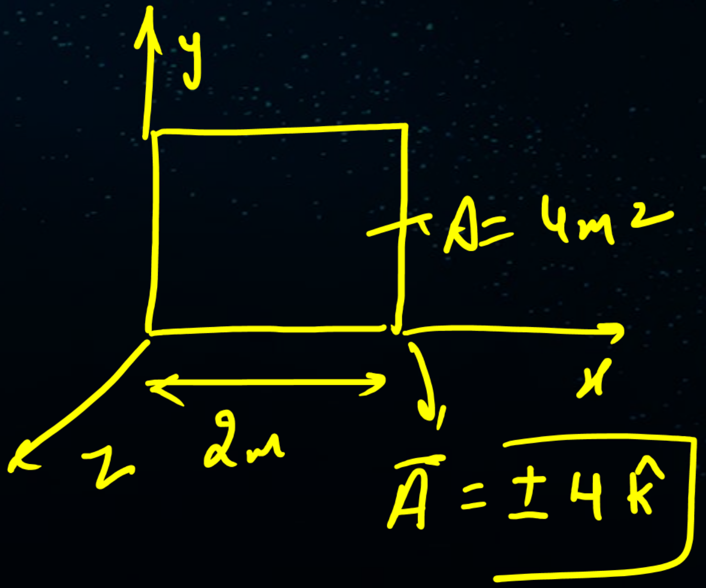
PROPERTIES OF ELECTRIC FIELD LINES | Recall

- Electric field lines always begin from a positive charge or infinity and terminate on a negative charge or infinity.
- Tangent at any point on electric field line gives the direction of electric field and electric force at that point i.e., known as electric lines of forces.
- Electric field lines always represents net electric field.
- Two electric field lines never intersects each other because at intersection point there are two possible direction of electric field which is not possible.
- If electric field line density is high, then intensity of electric field is high and vice versa.
- Electric field lines never exists in closed loop because it is a conservative force field therefore work done in closed loop should be zero, which is not possible.
- Number of electric field lines coming out and going in $\propto$ Magnitude of charge.

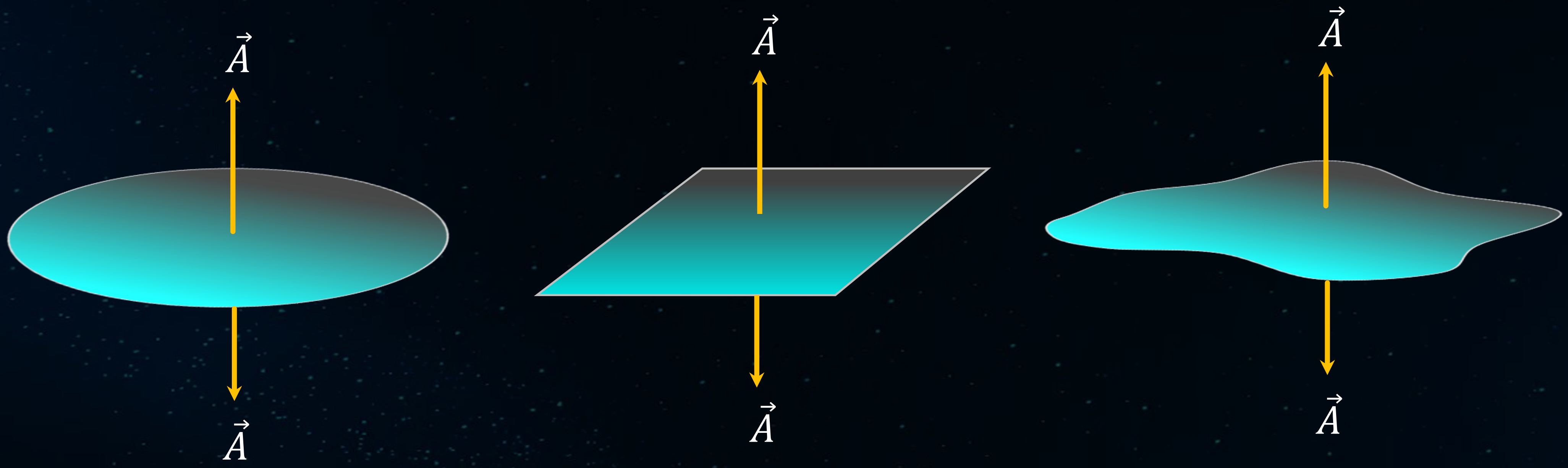
AREA VECTOR

- Direction of Area Vector is always normal to the surface.
- Vector Quantity.
- SI Unit : m^2
- Consider one area vector direction as positive and the opposite one as negative.





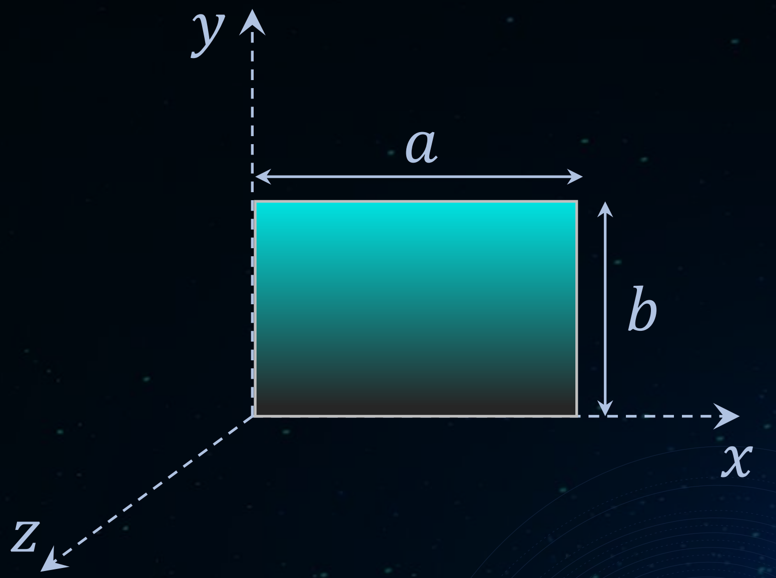
- All the 2 – D surfaces are considered as open surfaces.



- Only one direction of area vector of an open surface is considered for a particular problem.

A rectangle of length a and width b is placed in an $x-y$ plane as shown. Find the area vector of the rectangle.

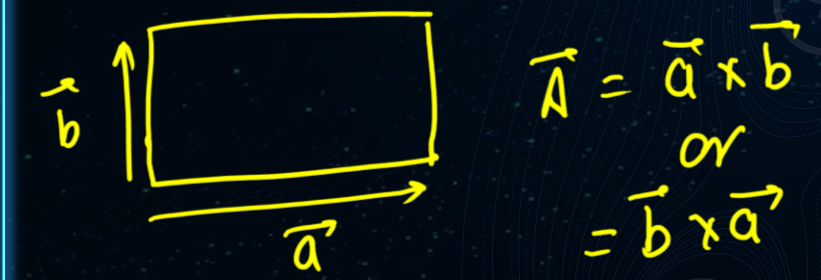
Solution



The magnitude of area of a rectangle of length a and width b is, $A = ab$

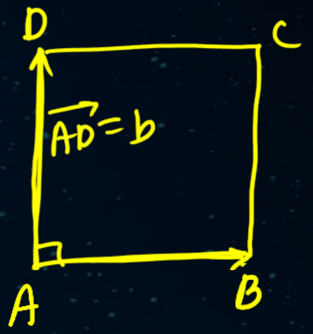
Since the rectangle is on xy-plane, the perpendicular vector of the rectangle will be along the z-axis. Thus, the area vector of the rectangle is, $\vec{A} = \pm ab\,\hat{k}$

$$\vec{A} = \pm ab\,(\hat{k})$$



A square of side 5 m is placed as shown. Find the area vector of the rectangle.

Solution



Area of rectangle $\vec{A} = \vec{AB} \times \vec{AD}$

In this case,

$\vec{AD} = 5\hat{j}$

$\vec{AB} = 5\cos 37°\,\hat{\imath} + 5\cos 90°\,\hat{\jmath} + 5\sin 37°\,\hat{k} = 4\hat{i} + 3\hat{k}$

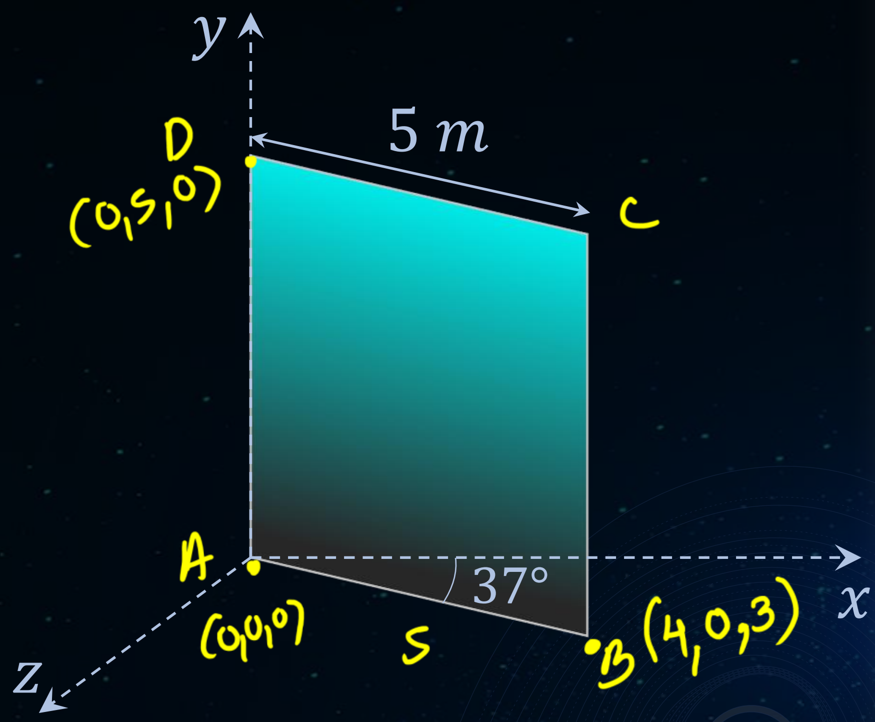
Therefore, the area vector will be:

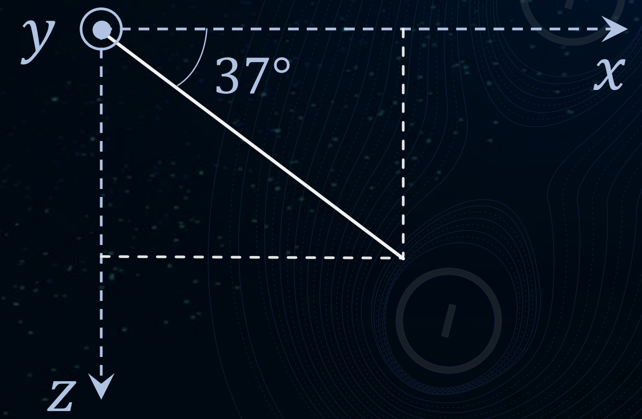
$\vec{A} = \vec{AB} \times \vec{AD}$

$= (4\hat{i} + 3\hat{k}) \times (5\hat{j})$

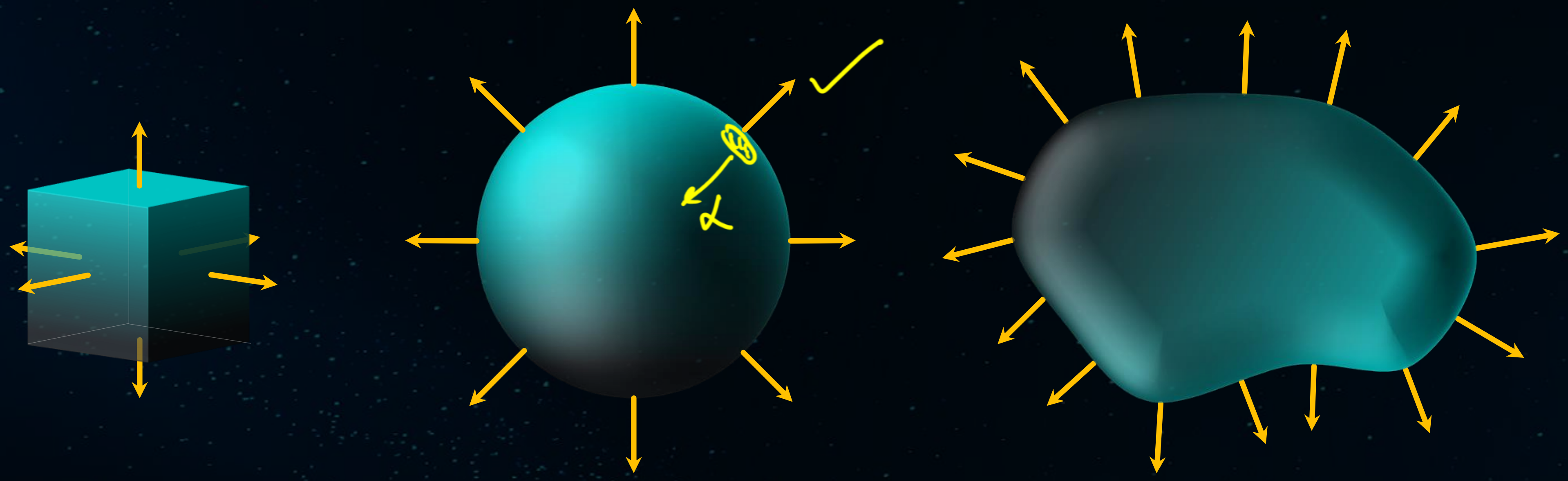
$= 20\hat{k} - 15\hat{i}$

$$\vec{A} = -15\,(\hat{\imath}) + 20\,(\hat{k})$$





- All the 3 – D surfaces are considered as Closed Surfaces.



- In closed surfaces, the direction of the area vector is always perpendicular to the surface but the normal should be in outward direction.

FLUX

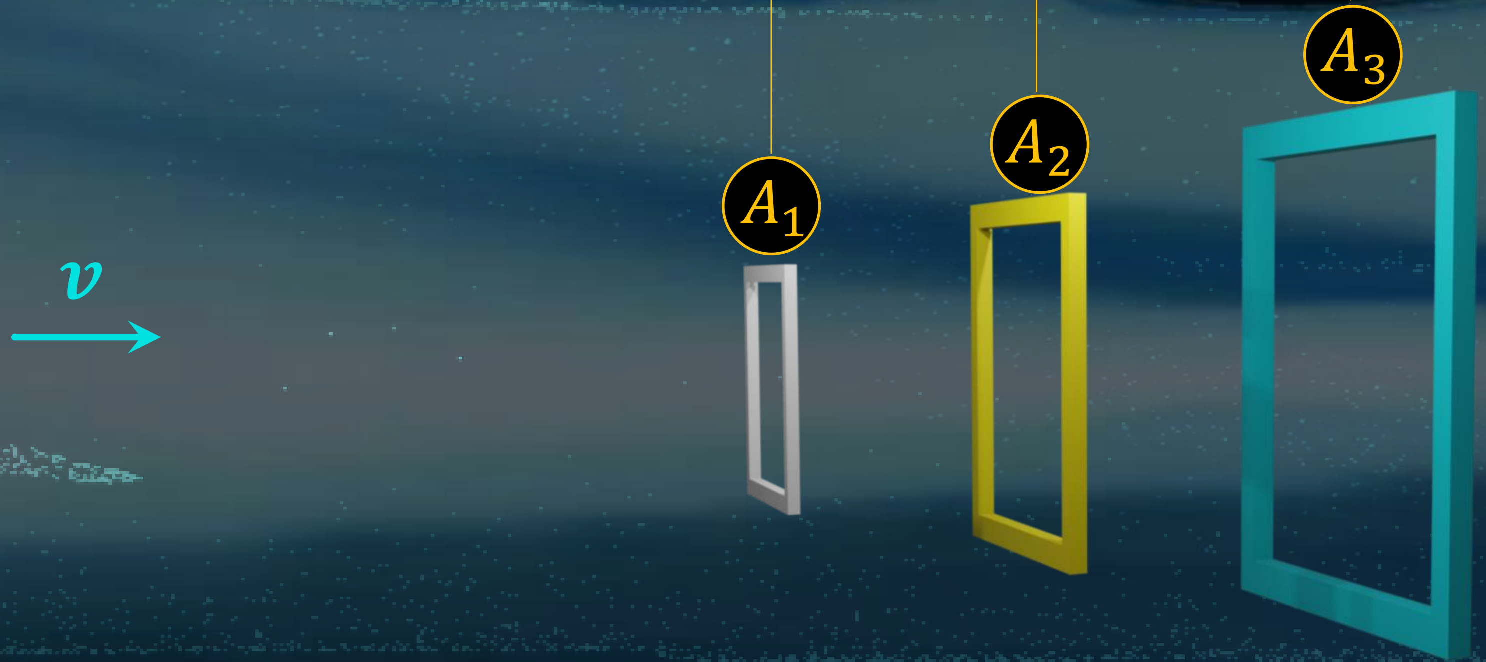
It is a flow of a quantity through an area normal to it.

It gives an idea about the amount of energy crossing an area or idea about the number of lines crossing an area normal to it.

For liquid, the greater the velocity the more will be the flux. Similarly, the greater the area the more will be the flux.

Analogy

Flux of liquid = vA_1 vA_2 vA_3

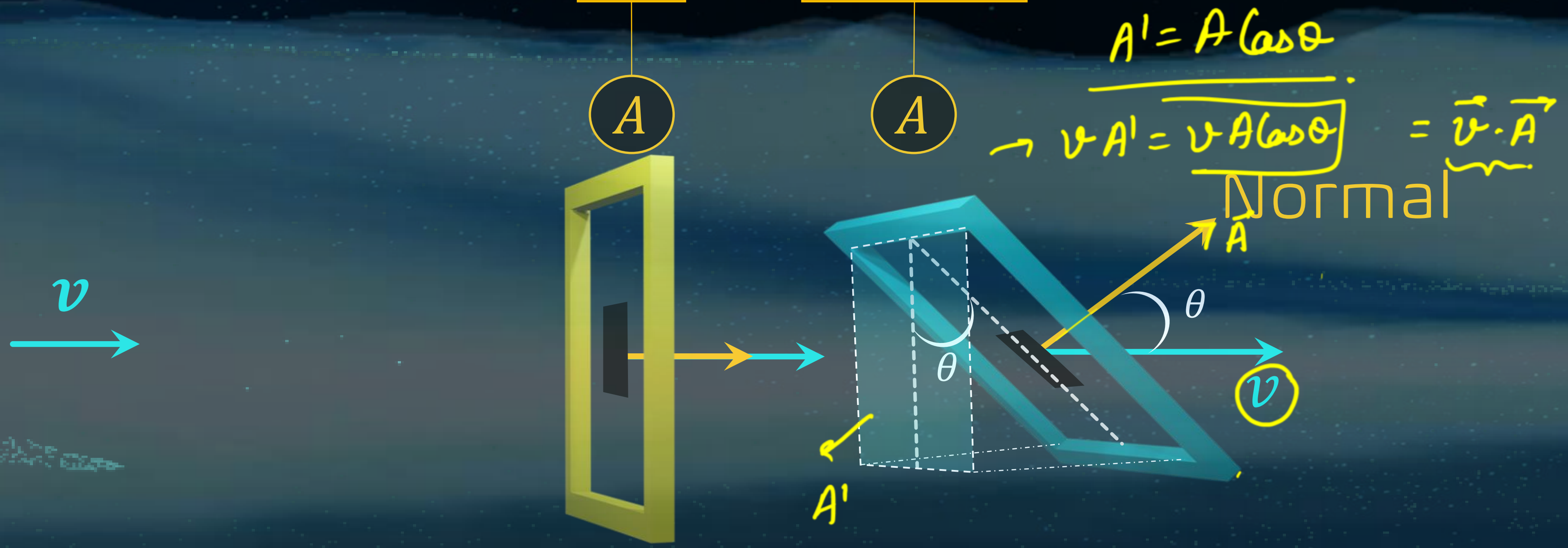


FLUX

Flux of liquid = $vA_{\perp}$

Where $A_{\perp}$ is the component of area which is perpendicular to the flow of the liquid

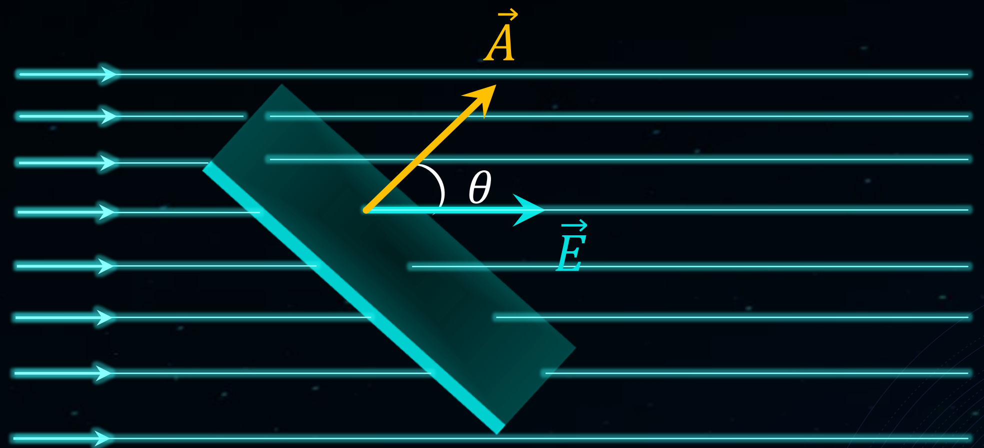
Flux of liquid = vA $vA\cos\theta$



ELECTRIC FLUX

BOARDS

It is the measure of the net electric lines of force crossing a surface normal to it.



»» SI Unit : $\frac{Nm^2}{C}$

For uniform electric field:

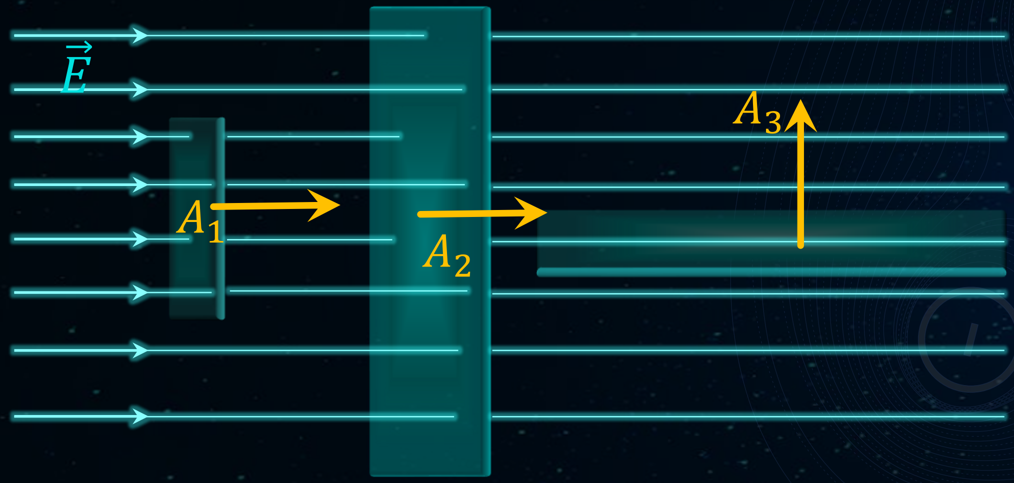
The electric flux is given by,

$$\phi = \vec{E} \cdot \vec{A}$$

For non-uniform electric field:

The electric flux is given by,

$$\phi = \int \vec{E} \cdot d\vec{A}$$

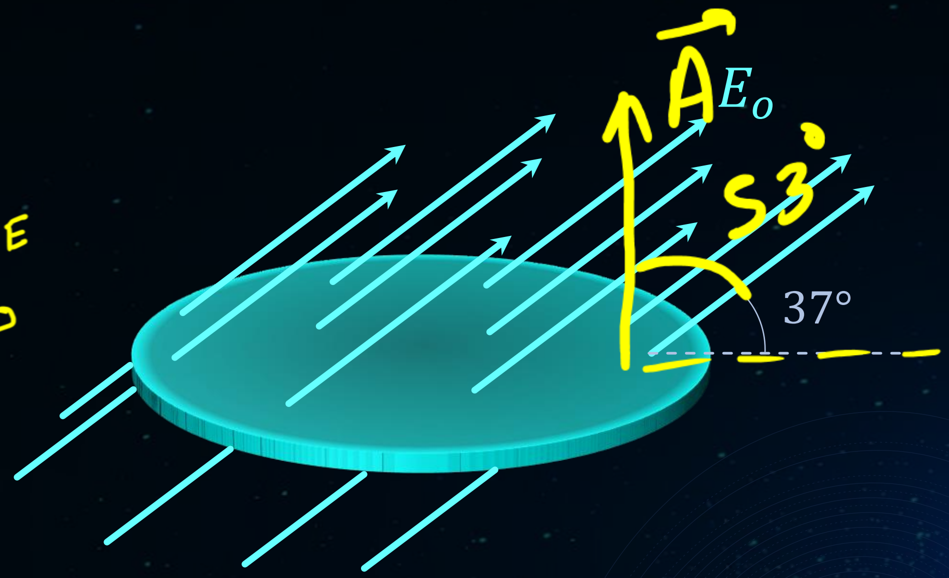


For the given figure, the electric field is uniform.

- Electric flux through the area A_1: $\phi_1 = \vec{E} \cdot \vec{A}_1 \Rightarrow \phi_1 = EA_1 \cos 0° = EA_1$
- Electric flux through the area A_2: $\phi_2 = \vec{E} \cdot \vec{A}_2 \Rightarrow \phi_2 = EA_2 \cos 0° = EA_2$
- Electric flux through the area A_3: $\phi_3 = \vec{E} \cdot \vec{A}_3 \Rightarrow \phi_3 = EA_3 \cos 90° = 0$

A circular disc of radius R is placed as shown. If the electric lines of force of strength E_o crosses the disc surface at $37°$, then find the electric flux through the surface of the disc.

Solution



The disc is a 2-D surface and hence, it is an open surface. Thus, there will be two possible directions of area vector.

Let us choose the direction of the area vector is along the upward direction. Therefore, the angle between the electric field line and the area vector is,

$(90° - 37°) = 53°$

In this case, the field lines are parallel. Hence, the electric field is uniform. Therefore, the electric flux through the surface of the disc is given by,

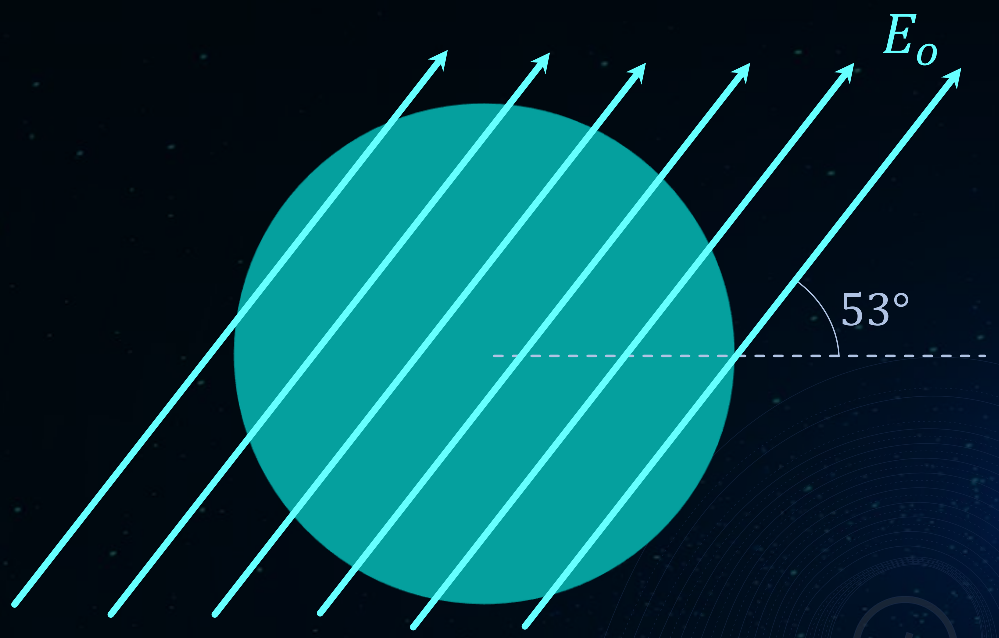
$$\phi = \vec{E} \cdot \vec{A} = E_O A \cos 53° = E_O(\pi R^2)\frac{3}{5}$$

$$\phi = \frac{3}{5} E_O(\pi R^2) \ \frac{Nm^2}{C}$$

$$\phi = \frac{3}{5} E_o \pi R^2$$

A circular disc of radius R is placed as shown. If the electric lines of force of strength E_o are going in the direction as shown, then find the electric flux through the surface of the disc.

Solution



According to the figure shown, the field lines are parallel to the surface of the disc. Hence, no field lines are able to cross the surface. Therefore, electric flux through the surface of the disc will be zero.

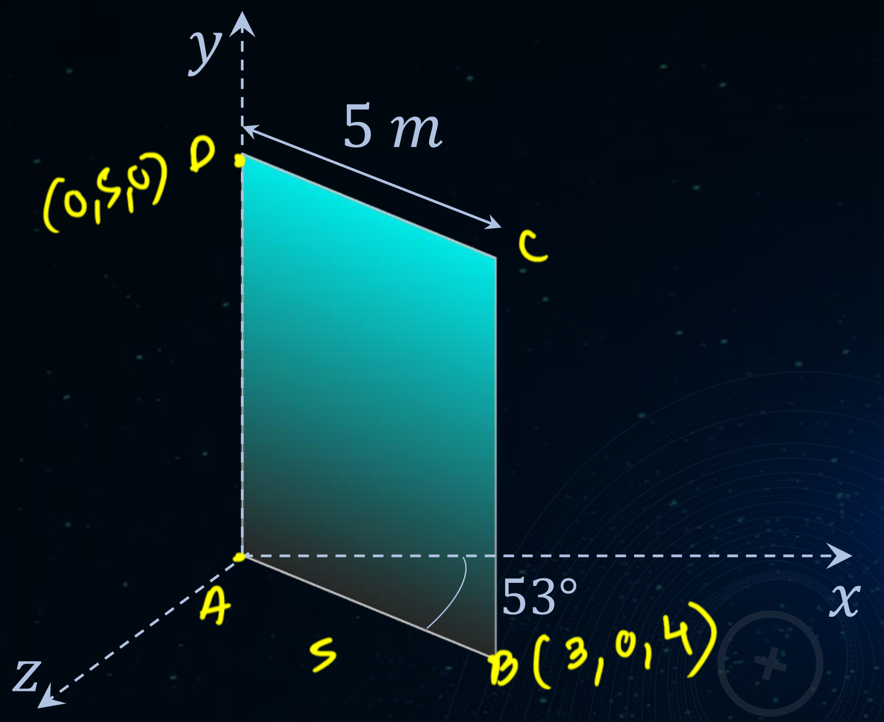
In other words, since the field lines are parallel to the surface of the disc, the area vector will be perpendicular the electric field lines. Therefore, electric flux through the surface of the disc will be,

$$\phi = \vec{E} \cdot \vec{A} = E_O A \cos 90° = 0$$

$$\phi = 0$$

A square of side $5\,m$ is placed as shown. If electric lines of force of strength $\vec{E} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ passes through the square, then find the electric flux through it.

Solution MAIN



In this case,

$\vec{AD} = 5\hat{j}$

$\vec{AB} = 5\cos 53°\,\hat{i} + 5\cos 90°\,\hat{j} + 5\sin 53°\,\hat{k} = (3\hat{i} + 4\hat{k})$

Therefore, the area vector will be:

$\vec{A} = \vec{AB} \times \vec{AD}$

$= (3\hat{i} + 4\hat{k}) \times 5\hat{j}$

$= (15\hat{k} - 20\hat{i})$

Hence, the electric flux through the surface is:

$\phi = \vec{E} \cdot \vec{A} = (2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (15\hat{k} - 20\hat{i})$

$= -40 + 60$

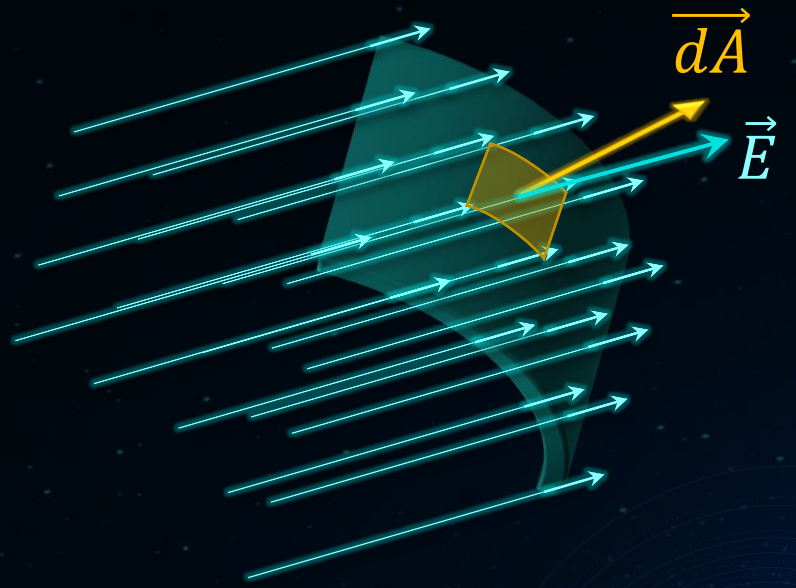
$= 20\ \frac{N\text{-}m^2}{C}$

$$\phi = 20\ \frac{Nm^2}{C}$$

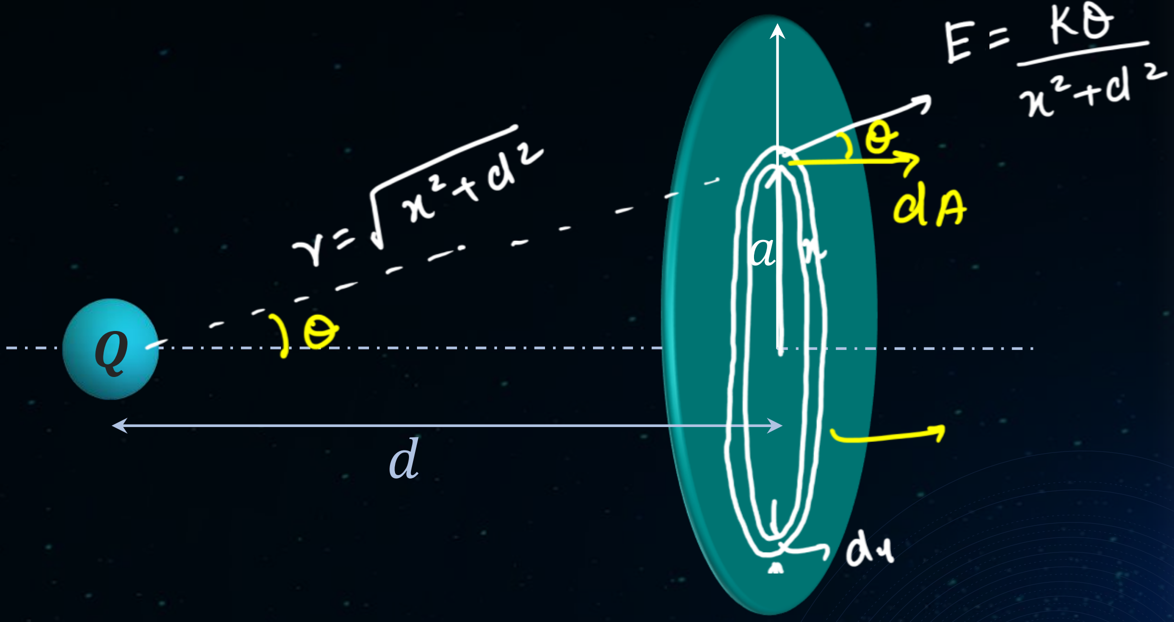
CURVED SURFACES

For curved surfaces, the angle between the electric field and the area vector changes at every point on the surface. Thus, the electric flux through the surface can be found as,

$$\phi = \int \vec{E}.\overrightarrow{dA}$$



A charge Q is placed on the axis of a disc of radius a at a distance d from the center of the disc as shown. Find the electric flux through the disc due to the charge Q.



Solution

Since the distance of Q from different points on the disc are different, the electric field on different points on the disc due to the point charge Q will also be different.

Consider an elementary ring of radius x and thickness dx. Therefore, the area of the ring is, $2\pi x \cdot dx = dA$

The distance of the charge Q from the top of the ring is,

$r = \sqrt{x^2 + d^2}$

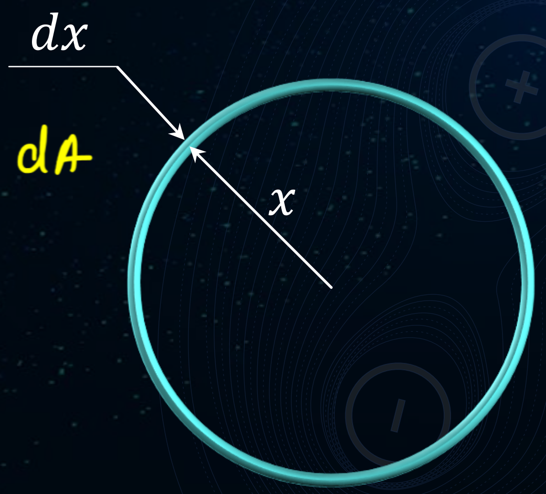
Therefore, the electric flux through the elementary ring due to the charge is given by,

$d\phi = E\, dA \cos\theta$

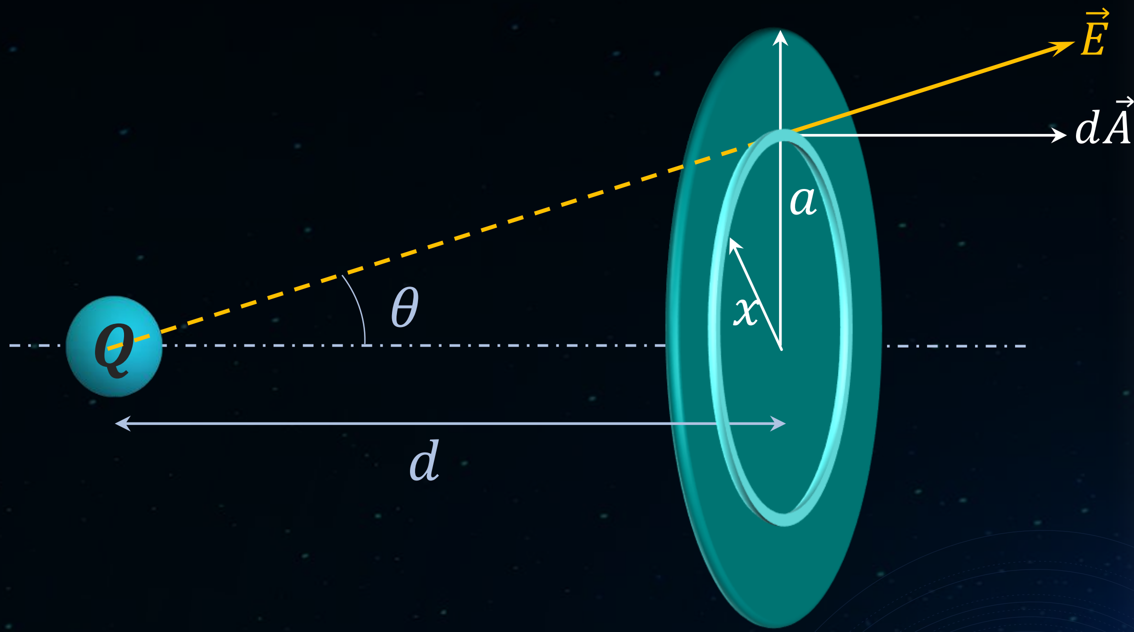
Now, $\cos\theta = \frac{d}{\sqrt{x^2+d^2}}$

Thus,

$d\phi = \frac{KQ}{(x^2+d^2)} 2\pi x\, dx \cdot \frac{d}{\sqrt{x^2+d^2}}$



Therefore, the total flux through the disc will be,



$$\phi_T = \frac{Q}{4\pi\varepsilon_0} 2\pi d \int_0^a \frac{x\,dx}{(x^2 + d^2)^{3/2}}$$

$$\phi_T = \frac{Qd}{2\varepsilon_0} \int_0^a \frac{x\,dx}{(x^2 + d^2)^{3/2}}$$

Let,

$$x^2 + d^2 = t^2$$

$$2x\,dx = 2t\,dt$$

$$x\,dx = t\,dt$$

For $x = 0,\ t = d$

For $x = d,\ t = \sqrt{d^2 + a^2}$

Therefore,

$$\phi_T = \frac{Qd}{2\varepsilon_0} \int_d^{\sqrt{d^2+a^2}} \frac{t\,dt}{t^3}$$

$$\phi_T = \frac{Qd}{2\varepsilon_0} \int_d^{\sqrt{d^2+a^2}} \frac{dt}{t^2}$$

$$\phi_T = \frac{Qd}{2\varepsilon_0} \left[-\frac{1}{t}\right]_d^{\sqrt{d^2+a^2}}$$

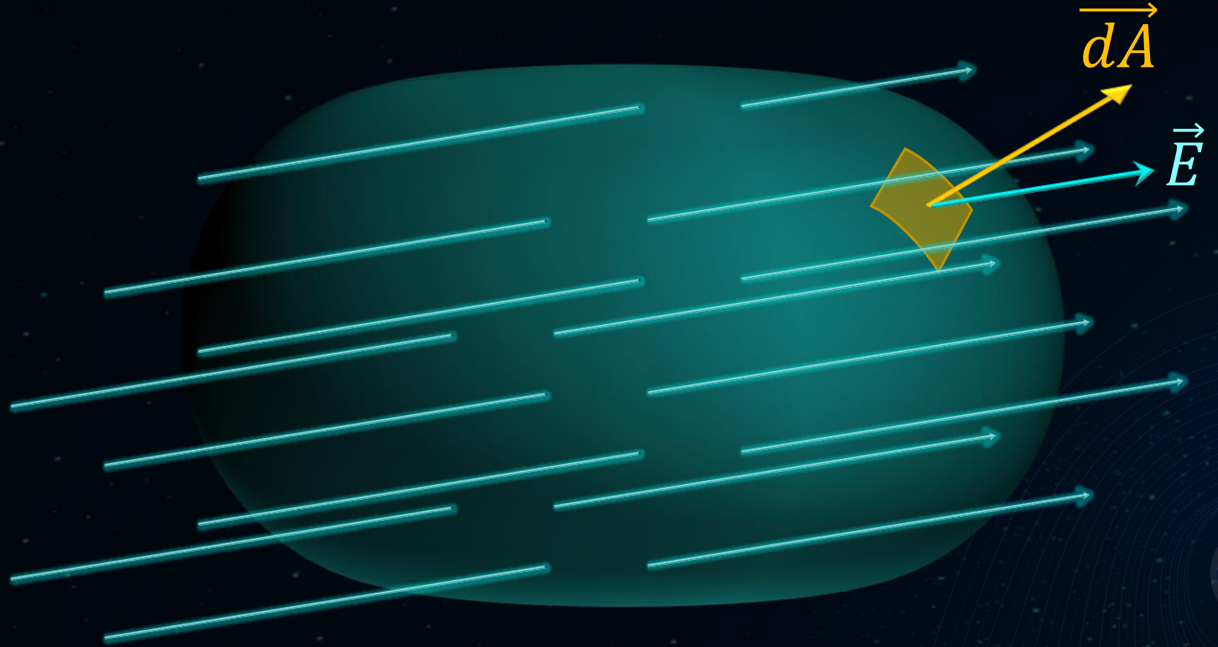
$$\phi_T = \frac{Qd}{2\varepsilon_0} \left[\frac{1}{d} - \frac{1}{\sqrt{d^2 + a^2}}\right]$$

$$\phi_T = \frac{Q}{2\varepsilon_0} \left[1 - \frac{d}{\sqrt{d^2 + a^2}}\right]$$

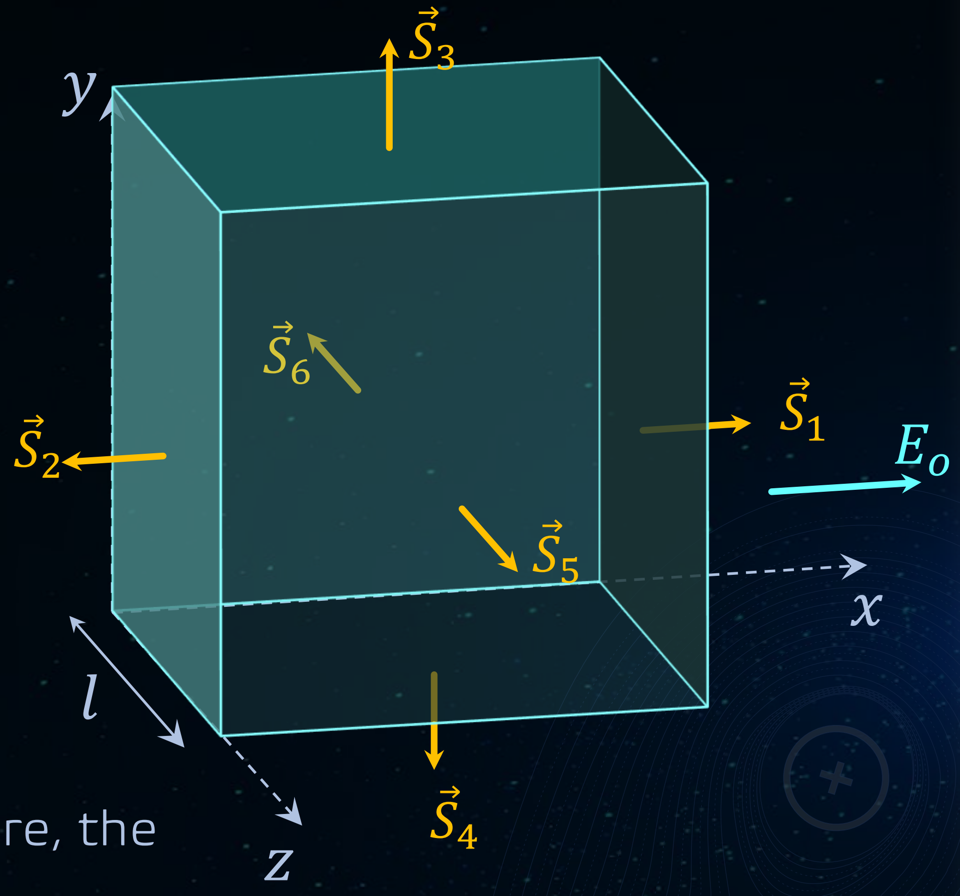
$$\phi = \frac{Q}{2\varepsilon_o}\left(1 - \frac{d}{\sqrt{d^2 + a^2}}\right)$$

For closed surfaces, the electric flux through the surface can be found as,

$$\phi = \oint \vec{E}.\overrightarrow{dA}$$



A cube of side l is placed as shown. If electric lines of force of strength $\vec{E} = E_o\,(\hat{\imath})$ passes through the cube, then find the net electric flux through it.



Solution

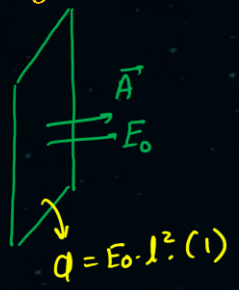
MAIN

The area vector $\vec{S}_3$, $\vec{S}_4$, $\vec{S}_5$ and $\vec{S}_6$ are perpendicular to the electric field $\vec{E}_o$. Hence, the electric flux due to these surfaces are zero.

The area vector $\vec{S}_1$ is parallel to the electric field $\vec{E}_o$. Therefore, the electric flux due to this surface will be:

$$\phi_1 = \vec{E} \cdot \vec{A}$$

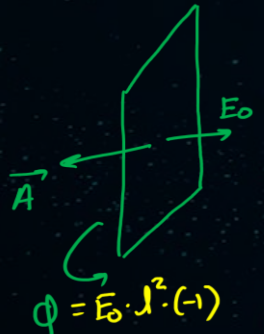
$$\Rightarrow \phi_1 = E_o A \cos 0° = E_o A = E_o(l^2)$$



The area vector $\vec{S}_2$ is anti-parallel to the electric field $\vec{E}_o$. Therefore, the electric flux due to this surface will be:

$$\phi_2 = \vec{E} \cdot \vec{A}$$

$$\Rightarrow \phi_2 = E_o A \cos 180° = -E_o A = -E_o(l^2)$$

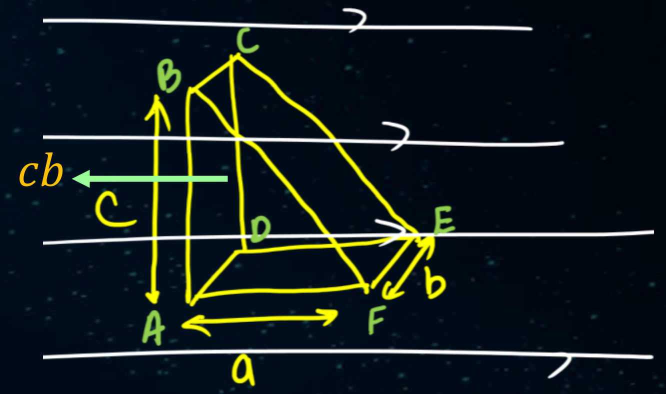


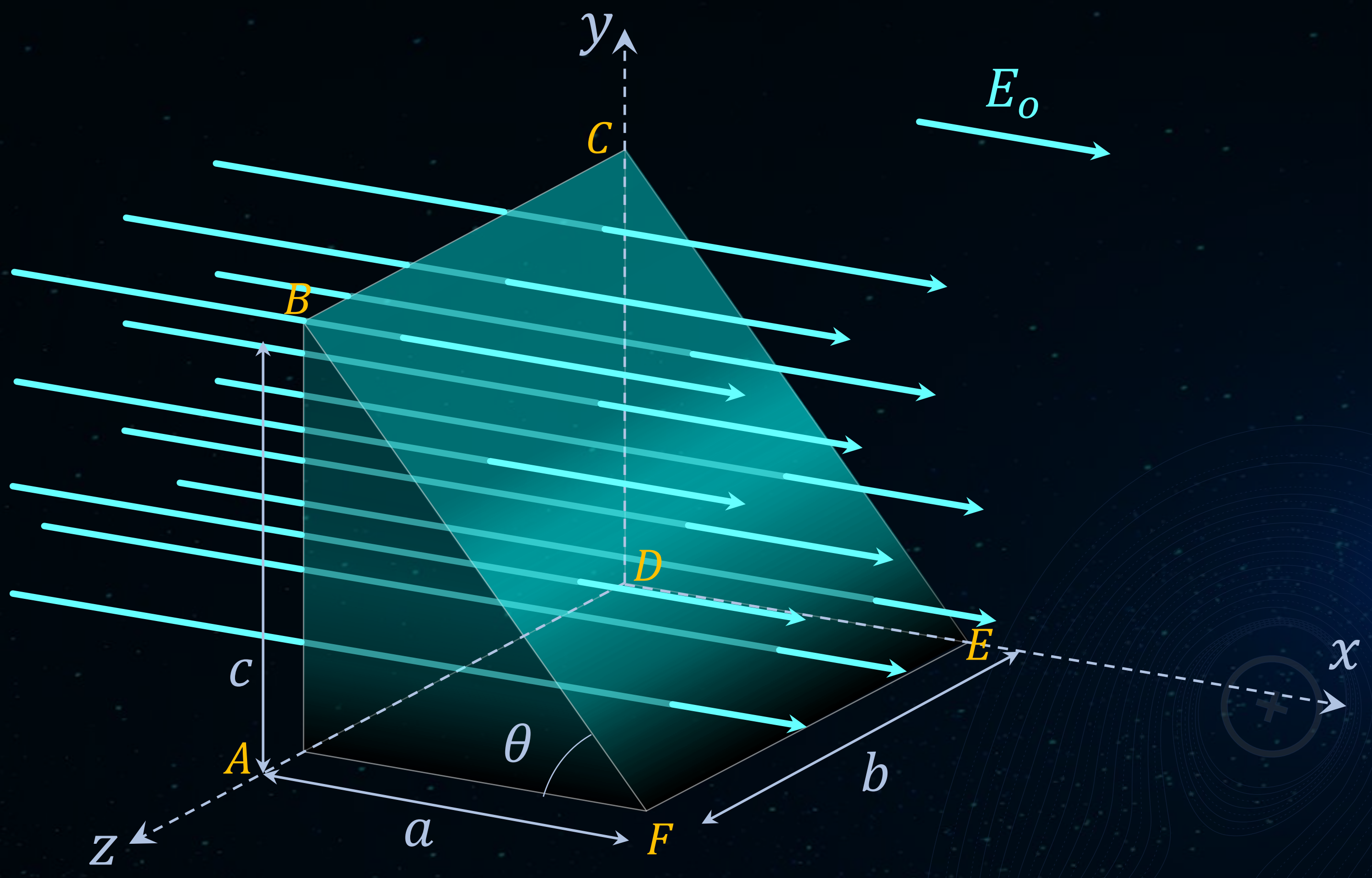
Therefore, the net electric flux $(\phi_1 + \phi_2)$ through the cube is zero.

$$\phi_{net} = 0$$

A prism shaped block is placed as shown. If electric lines of force of strength $\vec{E} = E_0\,\hat{\imath}$ passes through the block, then find the **net electric flux** through it.

Solution



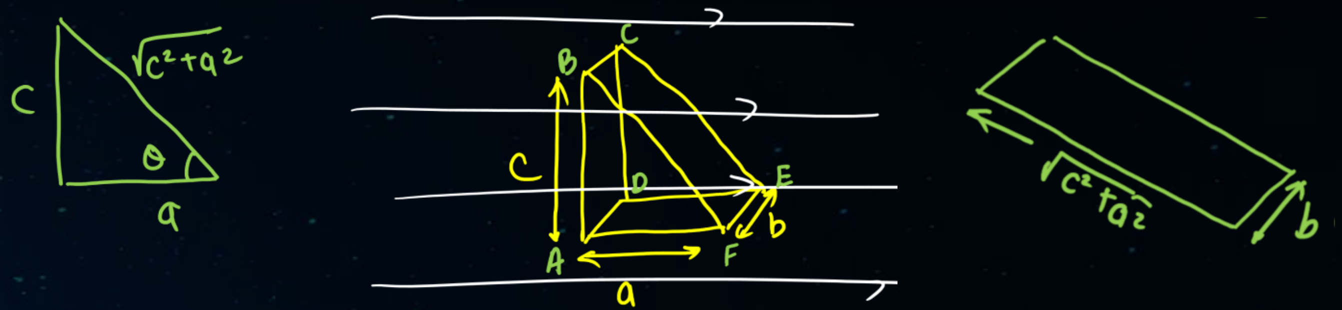


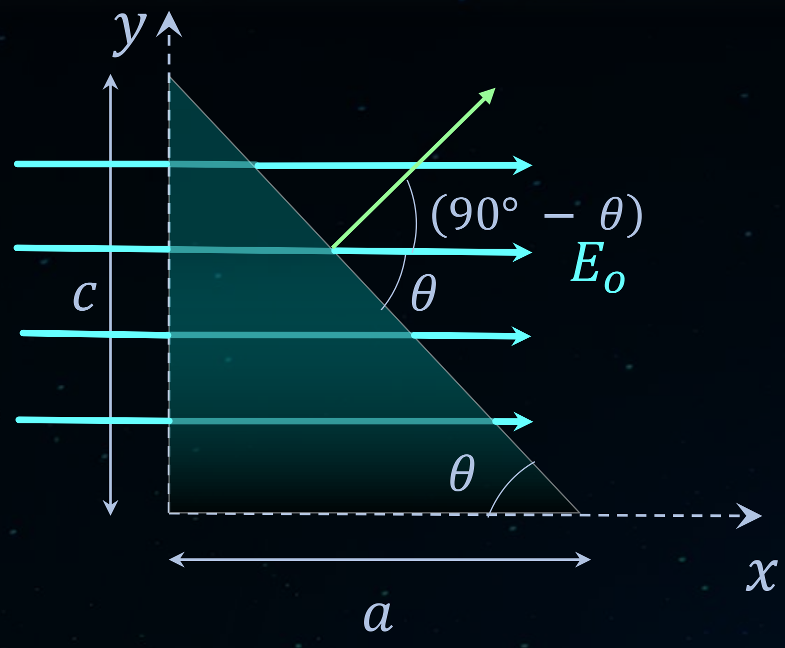
The area vector of the surface $ABCD$ is anti-parallel to the electric field $\vec{E}_o$. Therefore, the electric flux due to this surface will be:

$$\phi_1 = \vec{E}\cdot\vec{A}$$

$$\Rightarrow \phi_1 = E_o A\cos 180° = -E_o A = -E(cb)$$

Since the surface ABF, CDF and $ADEF$ are parallel to the electric field $\vec{E}_o$ which means that the area vector of each of these surface will be perpendicular to the electric field. Therefore, the electric flux through these surfaces will be zero.





The magnitude of area of the surface *BCEF* is,

$$A = b\sqrt{c^2 + a^2}$$

Therefore, the electric flux due to this surface will be:

$$\phi = \vec{E} \cdot \vec{A}$$

$$\Rightarrow \phi = E_o b\sqrt{c^2 + a^2}\cos(90° - \theta)$$

$$\Rightarrow \phi = E_o b\sqrt{c^2 + a^2}\sin\theta$$

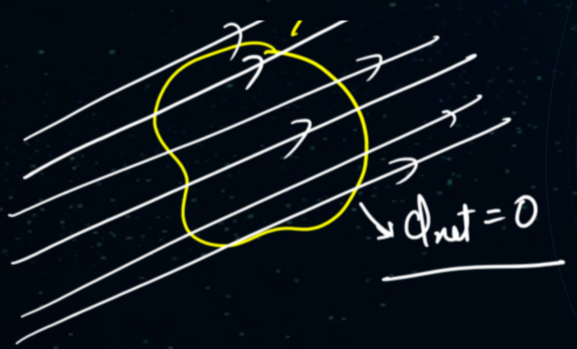
$$\Rightarrow \phi = E_o b\sqrt{c^2 + a^2} \times \frac{c}{\sqrt{c^2+a^2}}$$

$$\Rightarrow \phi = E_o(cb)$$

Therefore, the net electric flux through the cube is, $(\phi_1 + \phi) = 0$

$$\phi_{net} = 0$$

If $\vec{E}$ = Constant => Jitni lines closed Surface me Andar jayengi utni bahar aayengi. Matlab Net flux = 0.



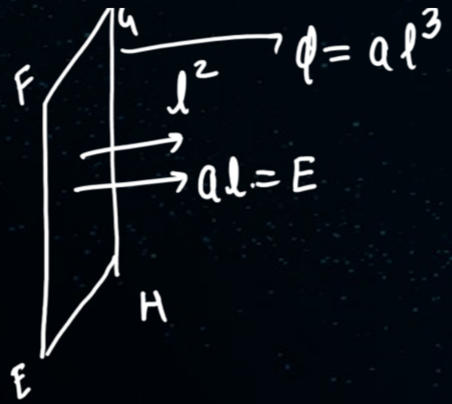
A square of side l is placed in electric lines of force of strength $\vec{E} = ax\ (\hat{\imath})$ passes through the cube, then find the electric flux through it.

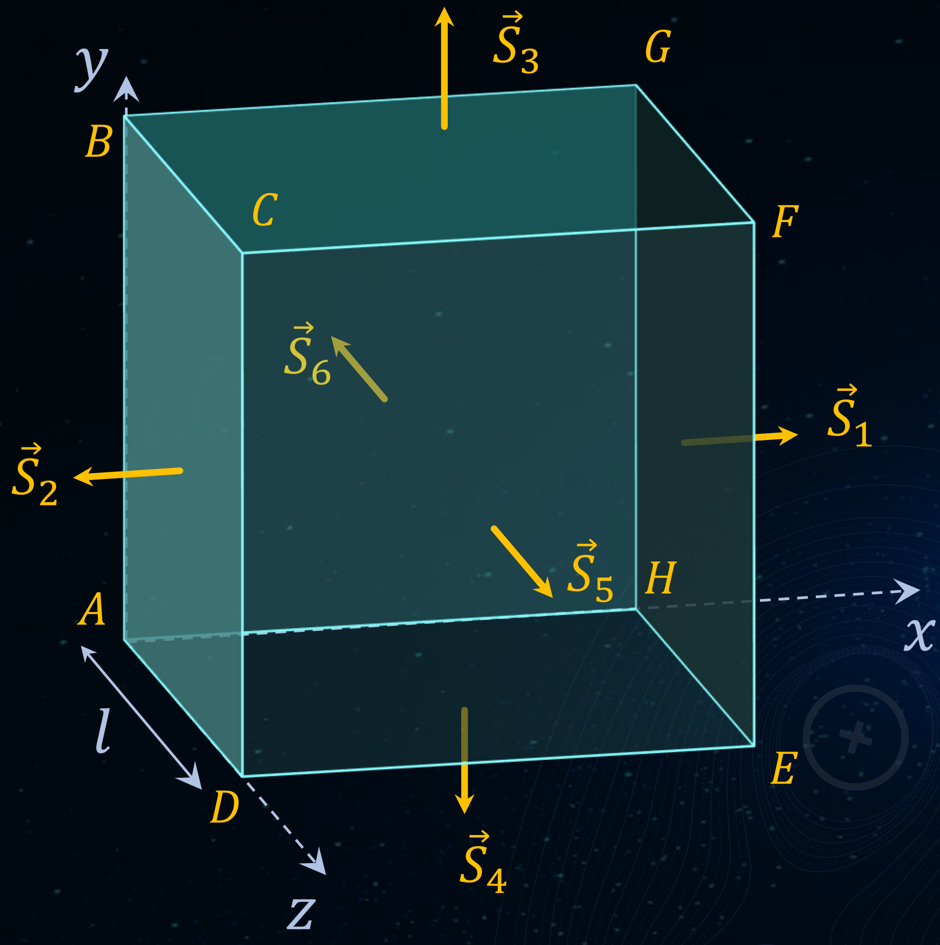
Solution

Since the electric field is $\vec{E} = ax\ (\hat{\imath})$, the surface $BCGF$, $CFED$, $DAHE$ and $ABGF$ are parallel to the electric field $\vec{E}$ which means that the area vector $\vec{S}_3$, $\vec{S}_4$, $\vec{S}_5$ and $\vec{S}_6$ will be perpendicular to the electric field. Therefore, the electric flux through these surfaces will be zero.

The surface $EFGH$ is located at $x = l$. Thus, the electric field will be, $\vec{E} = al\ (\hat{\imath})$. Now, the area vector of this surface i.e., $\vec{S}_1$ is parallel to the electric field $\vec{E}$. Therefore, the electric flux due to this surface will be:

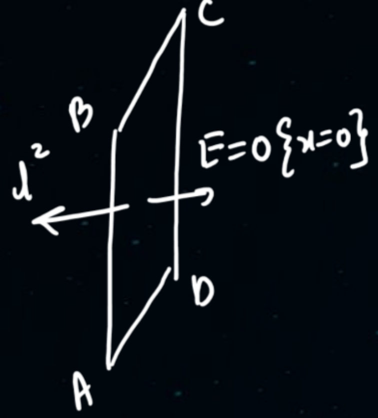
$$\phi_1 = \vec{E} \cdot \vec{A}$$
$$\Rightarrow \phi_1 = EA\cos 0^\circ = EA = al(l^2) = al^3$$

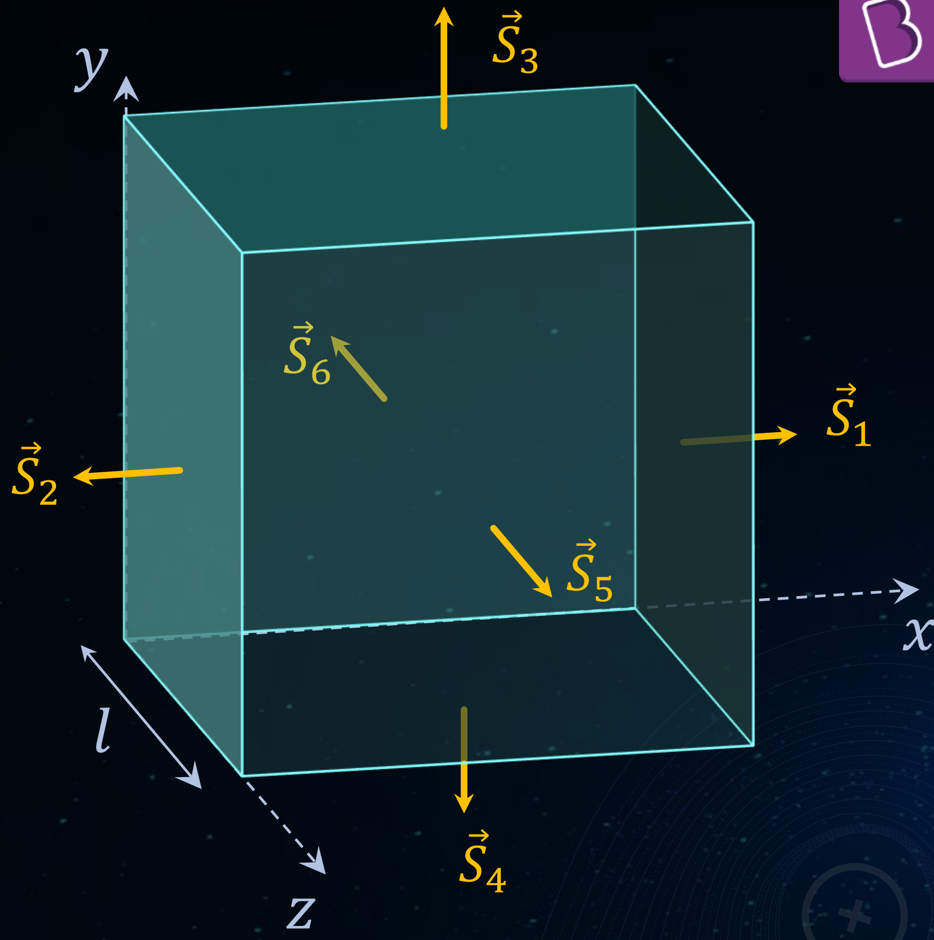




The surface $ABCD$ is located at $x = 0$. Thus, the electric field will be, $\vec{E} = 0\ (\hat{\imath})$. Now, the area vector of this surface i.e., $\vec{S}_2$ is anti-parallel to the electric field $\vec{E}$. Therefore, the electric flux due to this surface will be:

$$\phi_2 = \vec{E} \cdot \vec{A}$$
$$\Rightarrow \phi_2 = EA \cos 180° = 0$$





Therefore, the net electric flux through the cube is ,

$$\phi = \phi_1 + \phi_2 = al^3$$

$$\phi_{net} = al^3$$