WELCOME TO

BYJU'S CLASSES

ELECTROSTATICS

S21: Electric field, potential and potential energy due to Sphere

What you already know

- Electric field
- Electric potential
- Equipotential surface
- Electric dipole
- Calculation of electric field using Gauss's law

What you will learn

- Electric field, potential and potential energy due to Hollow Sphere
- Electric field, potential and potential energy due to Non-conducting solid sphere

चर्चा ON SPHERES

Solid Sphere

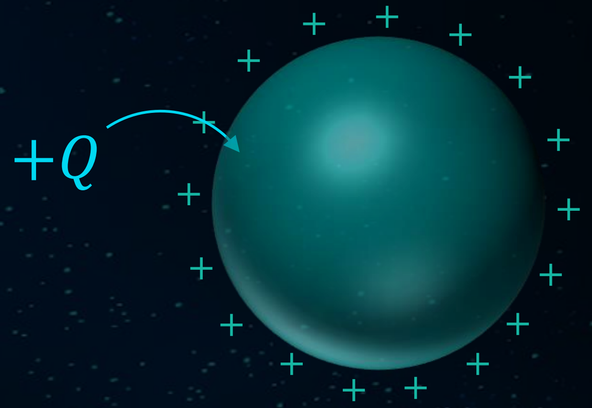
Hollow Sphere

Thick shell $(R_1 < R_2)$

Thin shell $(R_1 \approx R_2)$

Types of materials

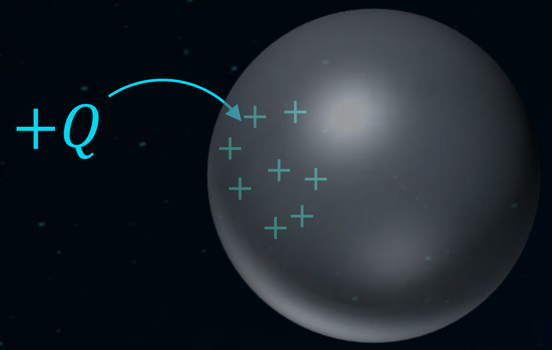
Conducting



Charge अपने आप को खुद arrange करेगा |

Electric field inside the conducting sphere is zero.

Non-conducting



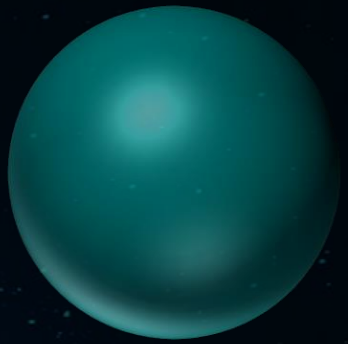
Charge जहां रख दोगे वही रह जायेगा |

चर्चा ON SPHERES

Spheres

- Hollow conducting Sphere (HC)

- Solid conducting Sphere (SC)

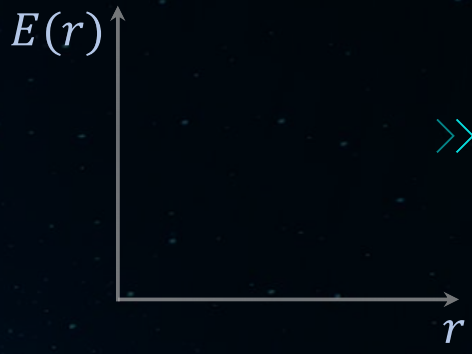


- Hollow non-conducting Sphere (HNC)

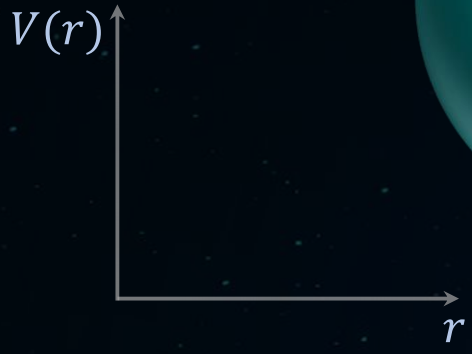
- Solid non-conducting Sphere (SNC)

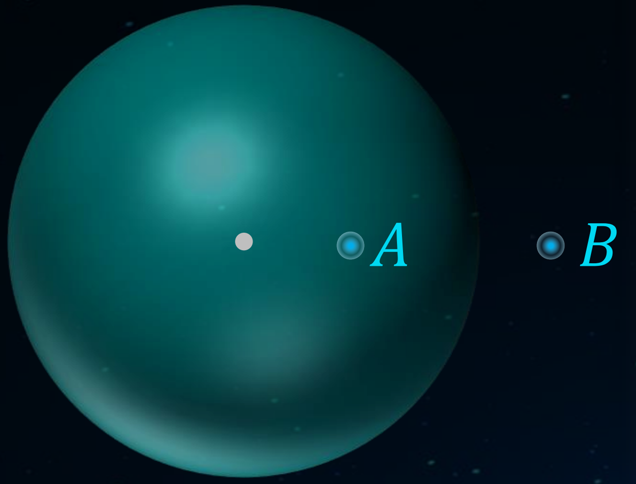
Points to be remember for every sphere

- E_A = Electric field at A
- E_B = Electric field at B
- Graph of $E(r)$ vs r



- U_T = Potential energy
- V_A = Potential at A
- V_B = Potential at B
- Graph of $V(r)$ vs r





Sphere

Electric field, potential, potential energy depends upon charge distribution.

चर्चा ON SPHERES

Spheres

- Hollow conducting Sphere (HC)
- Solid conducting Sphere (SC)
- Hollow non-conducting Sphere (HNC)
- Solid non-conducting Sphere (SNC)

If Q charge is put on these three spheres, the charge distribution on each of these spheres will be same.

Answer of all seven quantities mentioned in previous page will also be same.

E & V due to Hollow conducting, Solid conducting, Hollow non conducting Sphere

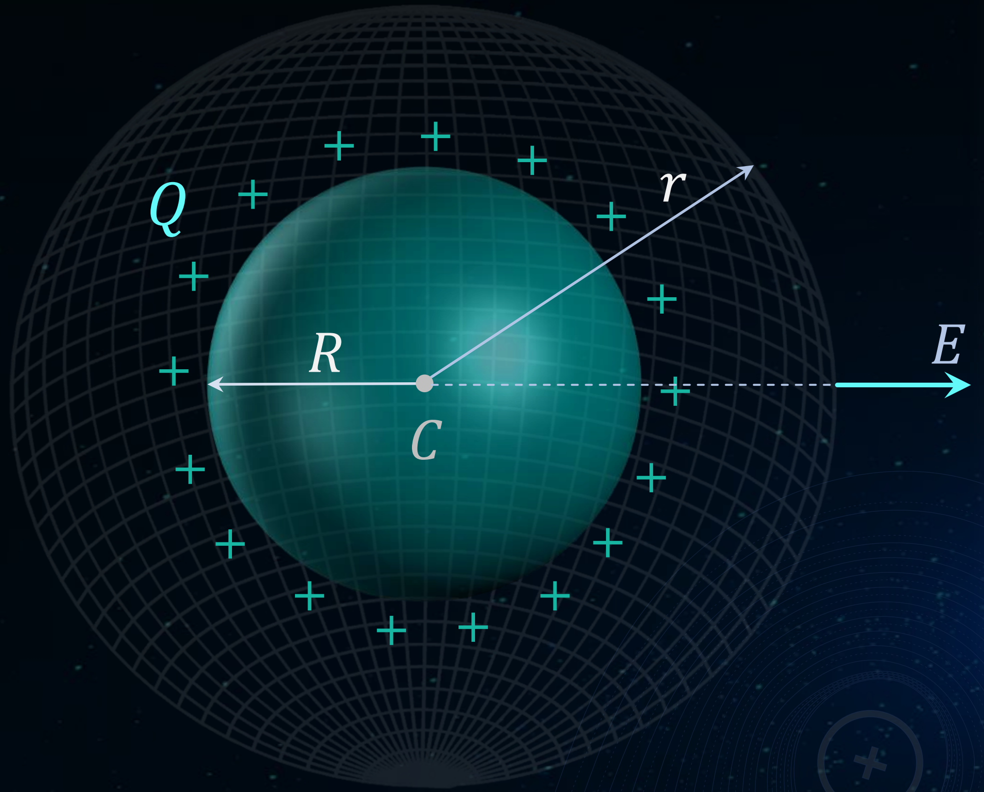
E & V at $r > R$

- For any point outside the sphere, it acts as a point charge placed at its centre.
- Therefore, the electric field at a point outside the sphere located r distance away from the centre of the sphere will be,

$$E = \frac{kQ}{r^2}$$

- The electric potential at the same point will be,

$$V = \frac{kQ}{r}$$



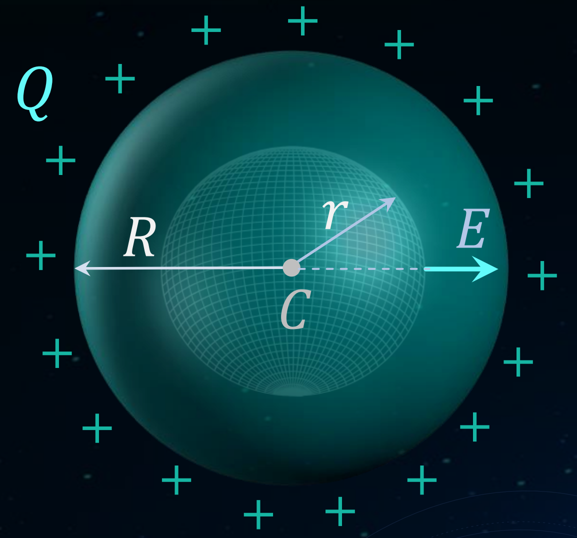
$$\boxed{E = \frac{kQ}{r^2}}$$

$$\boxed{V = \frac{kQ}{r}}$$

E & V due to Hollow conducting, Solid conducting, Hollow non conducting Sphere

E & V at $r < R$

- Because of this symmetry (spherical symmetry), consider a sphere (Gaussian surface) of radius r inside the sphere to find the electric field at a distance r from the centre of the sphere, as shown in the figure.
- Since all the charges are on the surface of the sphere, total charge enclosed by the gaussian surface is, $q_{in} = 0$
- The electric field $\vec{E}$ and any small area element $d\vec{A}$ are along same direction. Hence, the angle between them is, $\theta = 0°$



Applying Gauss's law, we get,

$$\oint \vec{E}\,.\,d\vec{A} = \frac{q_{in}}{\epsilon_0} \Rightarrow E(4\pi r^2) = 0 \Rightarrow E = 0$$

- Since the electric field inside the sphere is zero, the electric potential inside the sphere will be constant and the constant potential will be same as the potential at centre of the sphere. Thus, the potential will be:

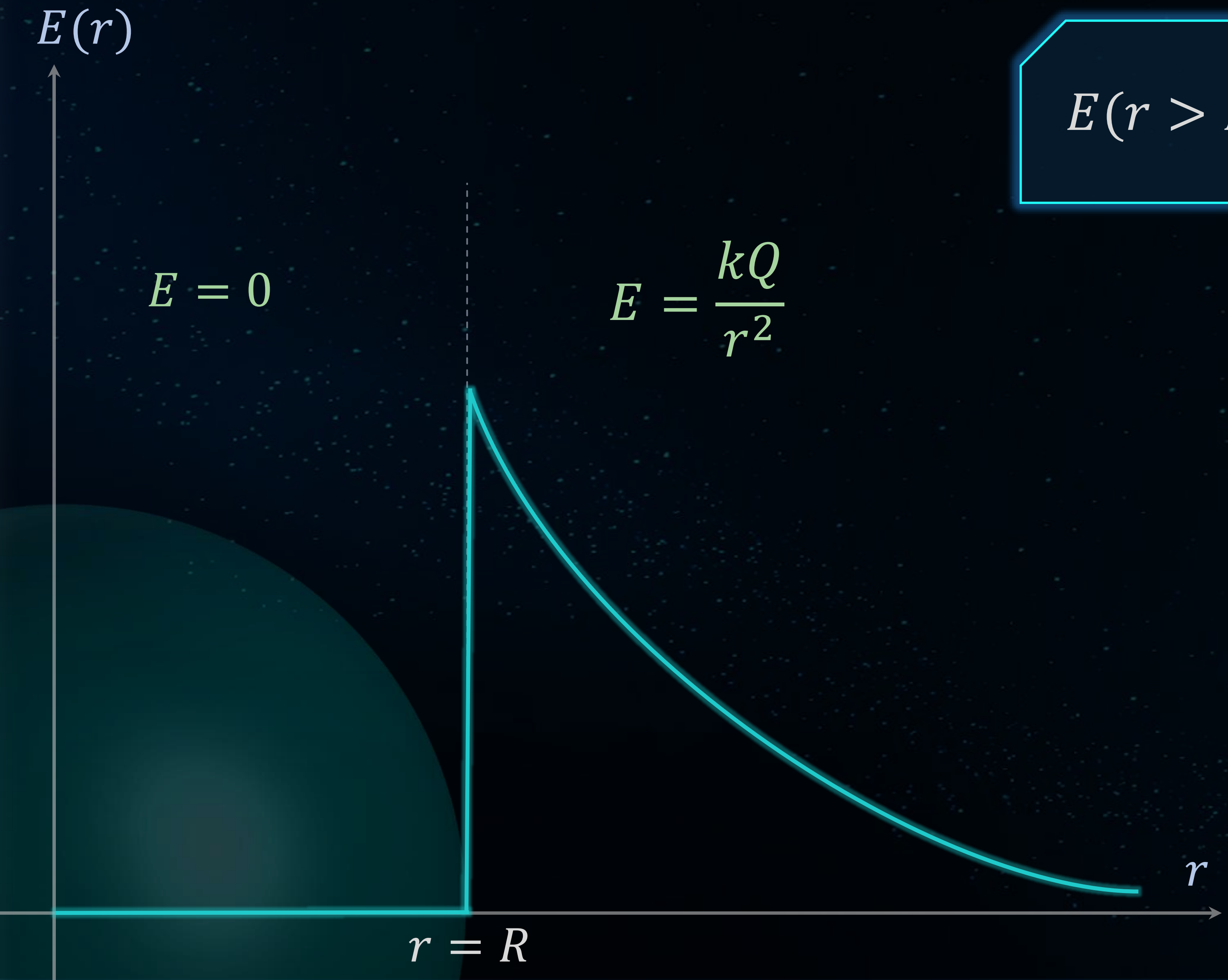
$$V(r) = V_c = \frac{kQ}{R}$$

$$\boxed{E = 0}$$

$$\boxed{V(r) = V_c = \frac{kQ}{R}}$$

E & V due to Hollow conducting, Solid conducting, Hollow non conducting Sphere

$E(r)$ vs r Graph

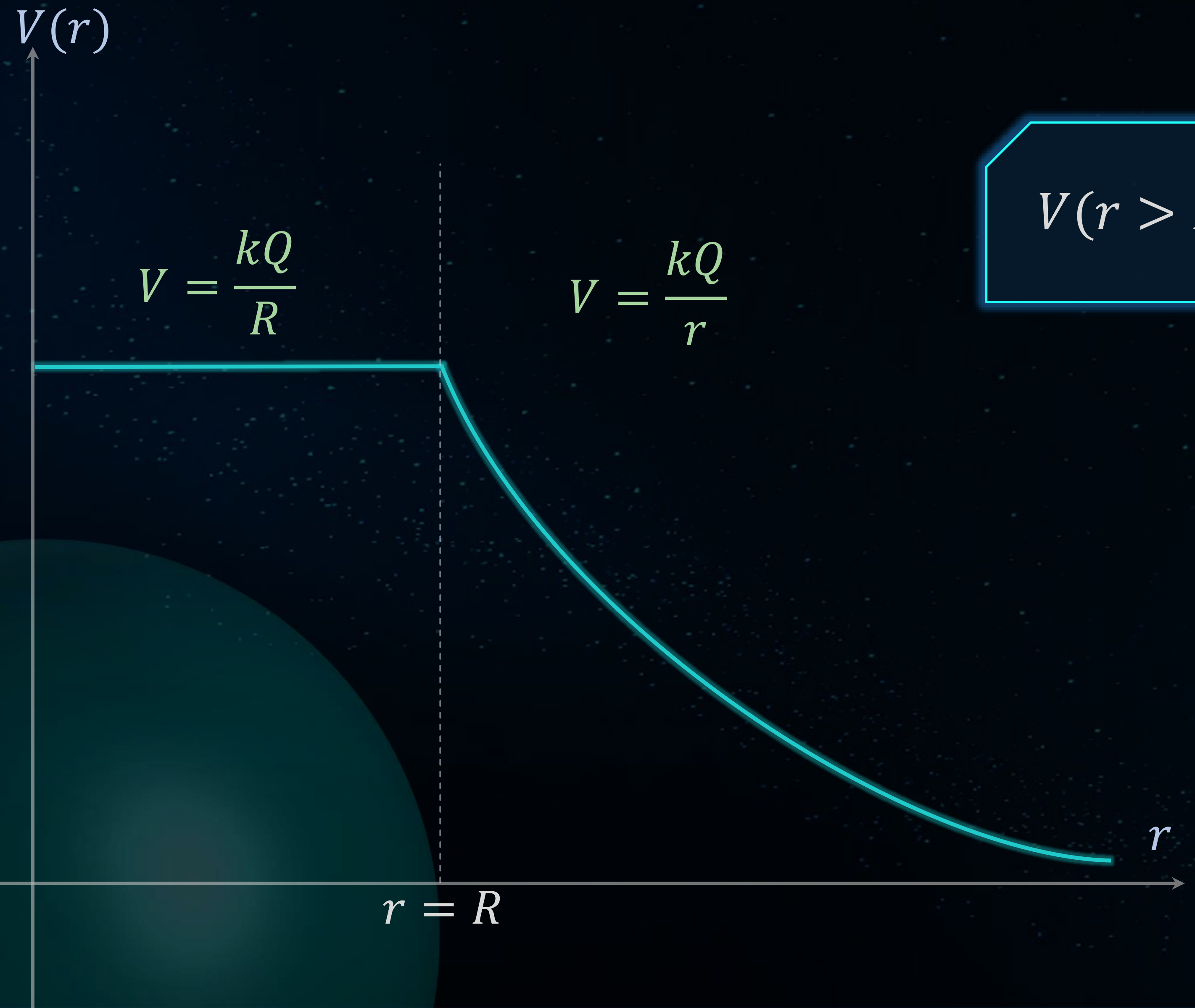


$$E(r > R) = \frac{kQ}{r^2}$$

$$E(r < R) = 0$$

E & V due to Hollow conducting, Solid conducting, Hollow non conducting Sphere

$V(r)$ vs r Graph



$$V(r > R) = \frac{kQ}{r}$$

$$V(r \leq R) = \frac{kQ}{R}$$

E & V due to Solid non conducting Sphere

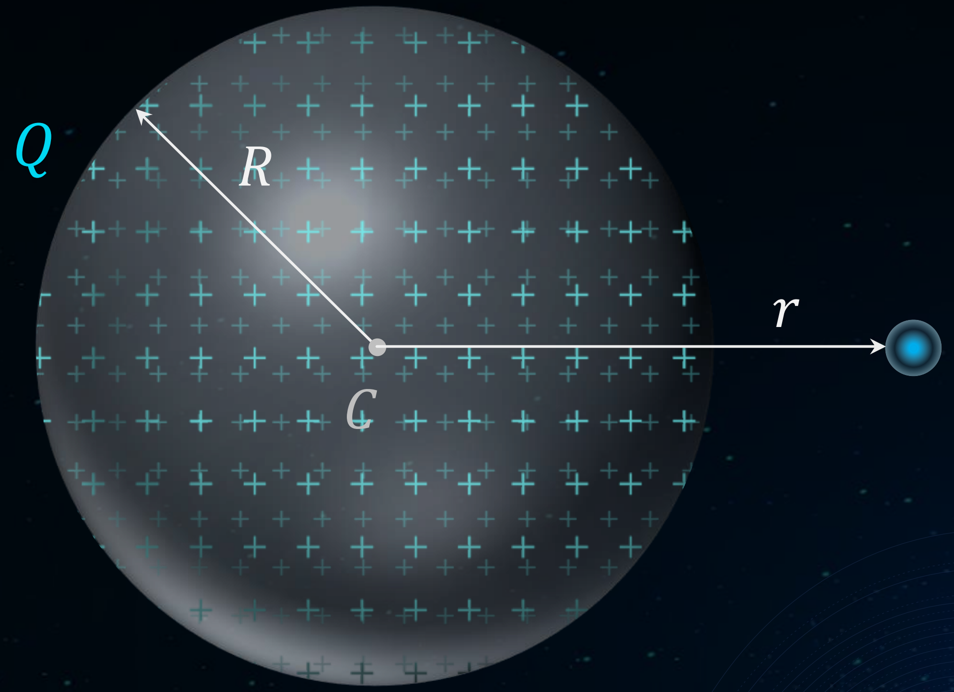
E & V at $r > R$

- For any point outside the sphere, it acts as a point charge placed at its centre.
- Therefore, the electric field at a point outside the sphere located r distance away from the centre of the sphere will be,

$$E = \frac{kQ}{r^2}$$

- The electric potential at the same point will be,

$$V = \frac{kQ}{r}$$



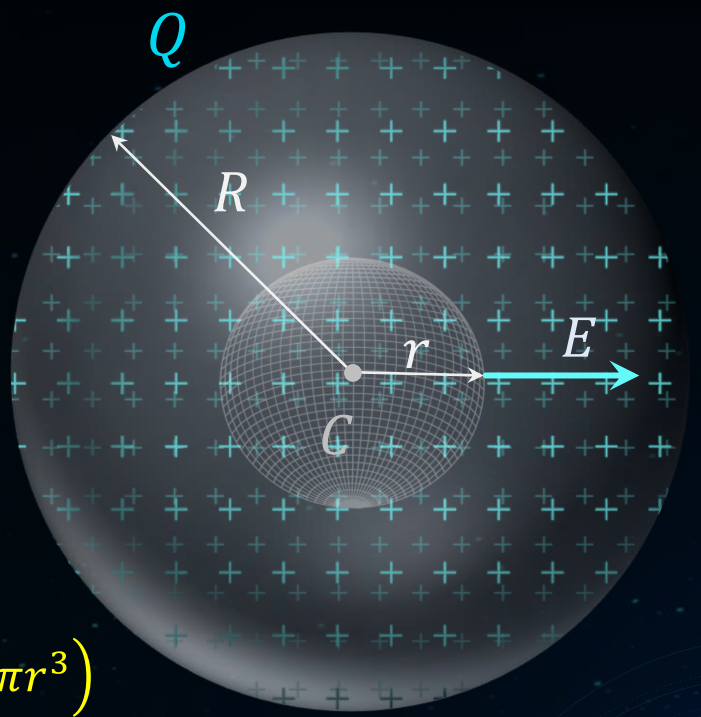
$$\boxed{E = \frac{kQ}{r^2}}$$

$$\boxed{V = \frac{kQ}{r}}$$

E & V due to Solid non conducting Sphere

E at $r < R$

- Since charge Q is distributed on a SNC sphere of radius R, the volume charge density of the sphere will be: $\rho = \frac{Q}{\left(\frac{4}{3}\pi R^3\right)}$
- Consider a sphere (Gaussian surface) of radius r inside the sphere to find the electric field at a distance r from the centre of the sphere, as shown in the figure.
- Since the charges are uniformly distributed throughout the whole sphere, total charge enclosed by the Gaussian surface is, $q_{in} = \rho\left(\frac{4}{3}\pi r^3\right)$



Applying Gauss's law, we get,

$$\oint \vec{E}\cdot d\vec{A} = q_{in}/\varepsilon_0$$

$$E\cdot 4\pi r^2 = \frac{\rho\cdot 4/3\,\pi r^3}{\varepsilon_0} = \frac{\frac{Q}{\cancel{4/3}\,\pi R^3}\cdot \cancel{4/3}\,\pi r^3}{\varepsilon_0}$$

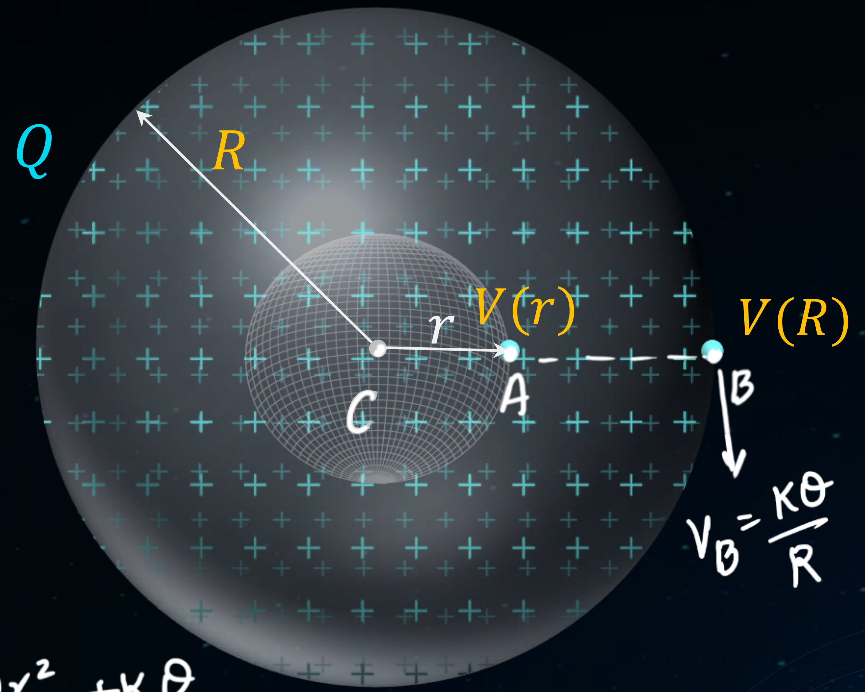
$$E\cdot 4\pi \cancel{r^2} = \frac{Q\cdot r^{\cancel{3}}}{R^3\varepsilon_0} \Rightarrow \boxed{E = \frac{KQr}{R^3} = \frac{\rho r}{3\varepsilon_0}}$$

$$\boxed{E = \frac{\rho r}{3\varepsilon_0} = \frac{kQr}{R^3}}$$

E & V due to Solid non conducting Sphere

V at $r < R$

- The potential on the surface of the conductor: $V_B = \frac{KQ}{R}$
- Suppose we want to find the potential at point A and potential at this point is assumed to be: $V_A = V(r)$
- We know that the electric field inside a SNC sphere of radius R and total charge Q is: $E = \frac{kQr}{R^3}$
- If we go from point A to point B, then the potential difference between these two points will be :



$$V_i - V_f = \int \vec{E}\cdot d\vec{r}$$

$$V_A - V_B = \int_r^R \frac{KQr}{R^3}\, dr$$

$$V(r) - \frac{KQ}{R} = \frac{KQ}{2R^3}\left[r^2\right]_r^R$$

$$V(r) = \frac{KQ}{2R^3}\left[R^2 - r^2\right] + \frac{KQ}{R}$$

$$V(r) = \frac{KQ}{2R} - \frac{KQr^2}{2R^3} + \frac{KQ}{R}$$

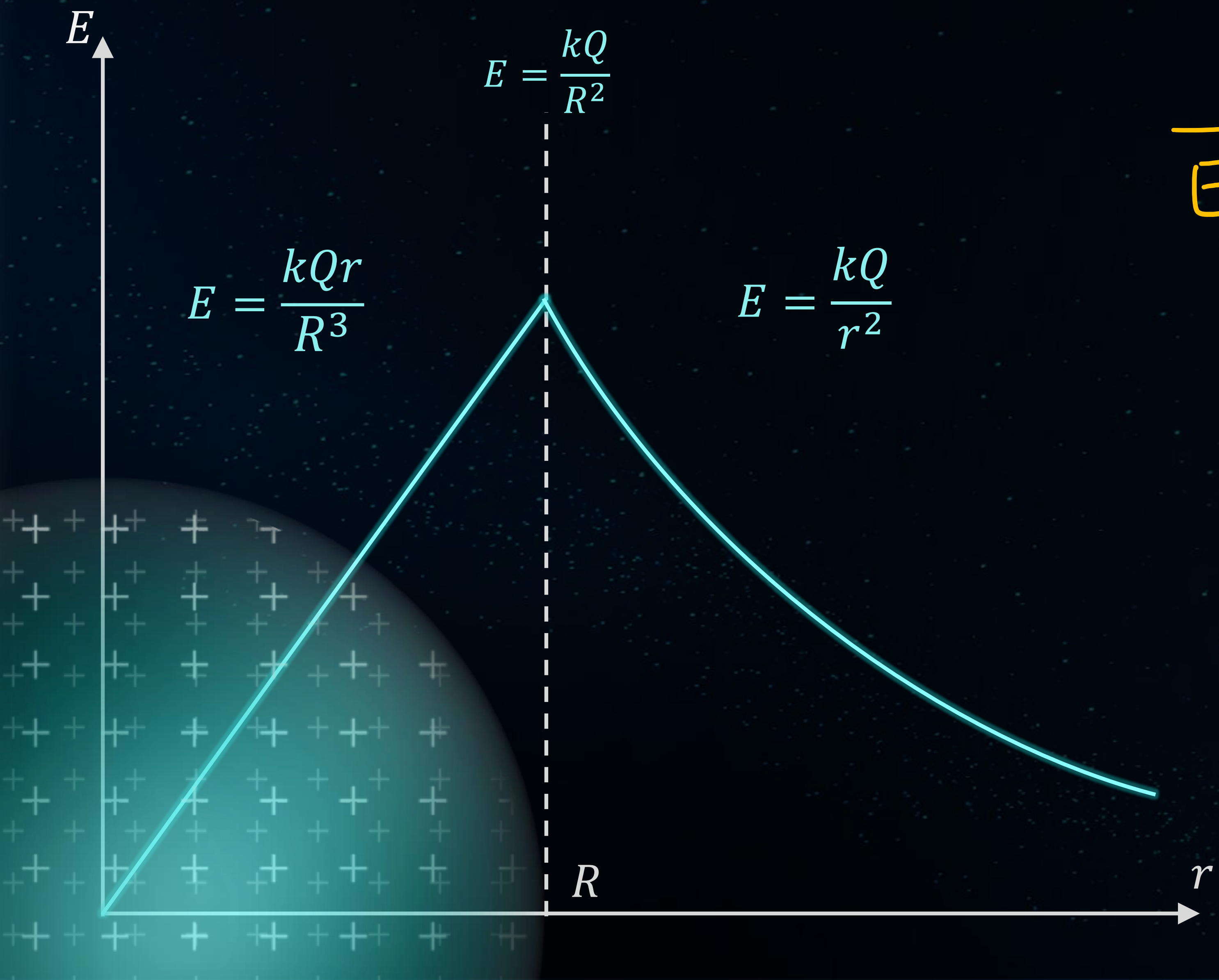
$$V(r) = \underbrace{\frac{3KQ}{2R}}_{a} - \frac{KQr^2}{2R^3} = \boxed{\frac{KQ}{2R^3}\left[3R^2 - r^2\right]}$$

$$V(r) = a - br^2$$ {Parabola Pani dhulta hua}

$$\boxed{V(r) = \frac{kQ}{2R^3}\left(3R^2 - r^2\right)}$$

E & V due to Solid non conducting Sphere

$E(r)$ vs r Graph

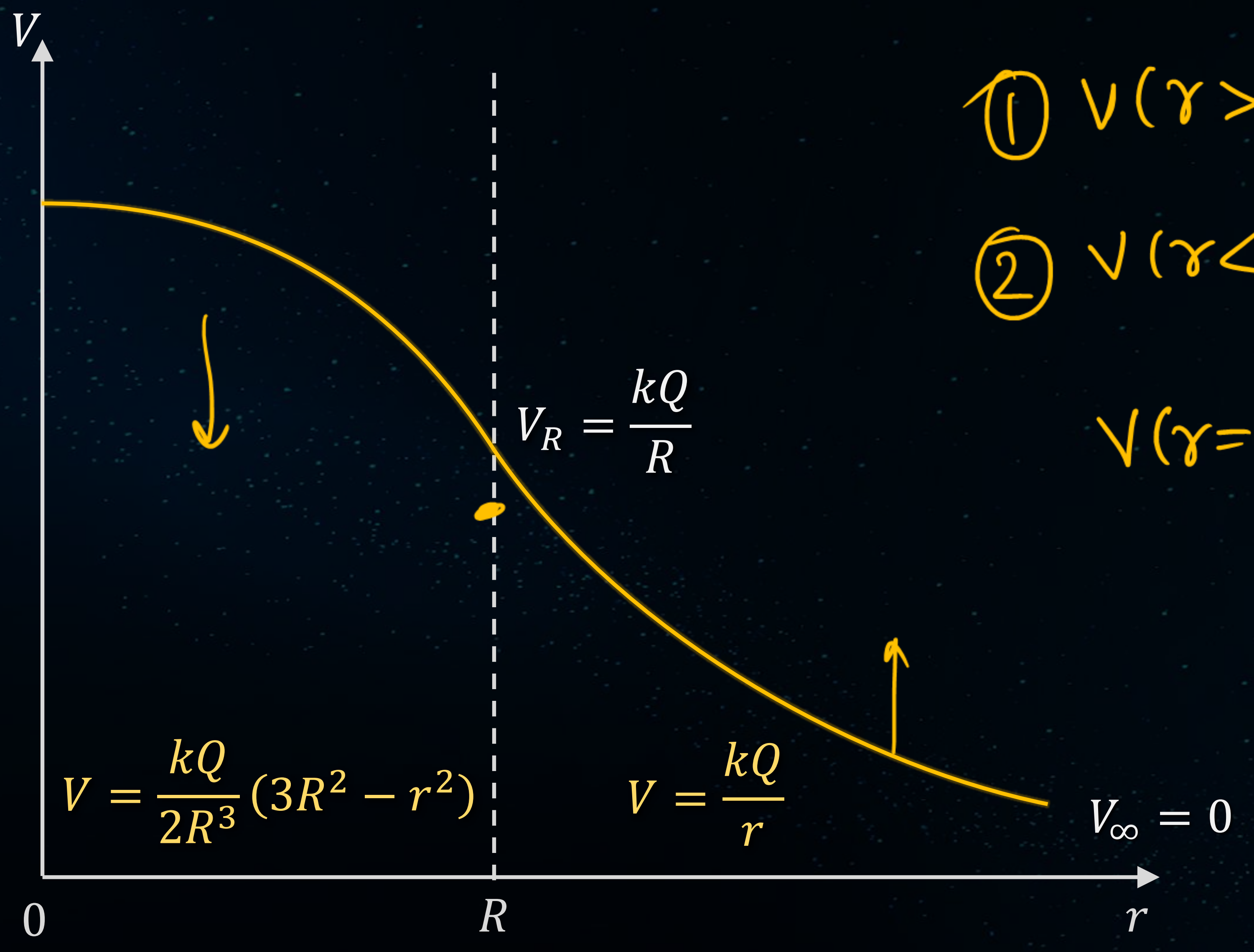


$$\vec{E}(r) \rightarrow \begin{cases} \frac{kQ\vec{r}}{R^3} & (r<R) \\ \frac{kQ}{R^2}\hat{r} & (r=R) \\ \frac{kQ}{r^2}\hat{r} & (r>R) \end{cases}$$

E & V due to Solid non conducting Sphere

BOARDS MAIN

$V(r)$ vs r Graph

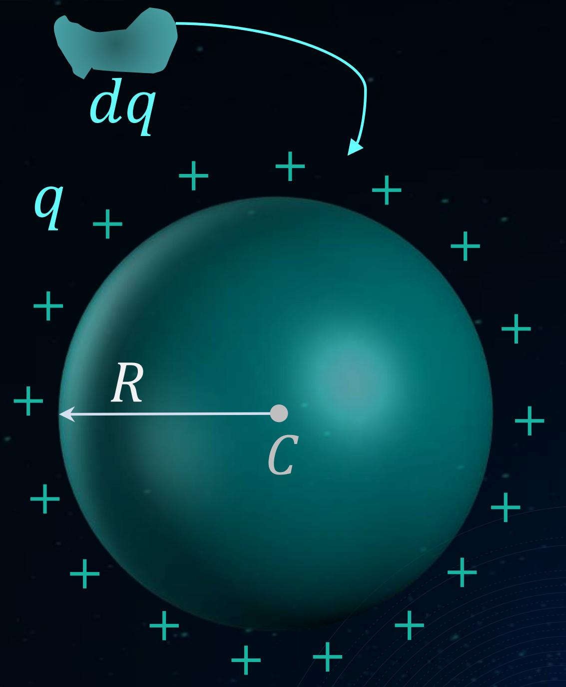


① $V(r>R) = \dfrac{KQ}{r}$

② $V(r<R) = \dfrac{KQ}{2R^3}(3R^2 - r^2)$

$V(r=0) = \dfrac{3KQ}{2R}$

U due to Hollow conducting, Solid conducting, Hollow non conducting Sphere



- Suppose, initially, we have an uncharged sphere. To charge the sphere, charges have to be brought externally from infinity to the given thin spherical conducting shell. At first, an external agent brings a charge $+q$ from infinity to the surface of the sphere. Since no charge is there on the surface of the sphere initially, it can be brought without doing work against the electric field.
- After this, to bring an elemental charge $+dq$ from infinity to the surface of the sphere, some external work has to be done against the electrostatic force and that work will be stored in the form of potential energy in the system.
- Initially, the sphere was uncharged, the potential of the sphere: $V_i = 0$
- When the sphere has charge $+q$, the potential of the sphere: $V_f = \frac{kq}{R}$
- Therefore, work done by the external agent to bring an elemental charge $+dq$ from infinity to the surface of the sphere is :

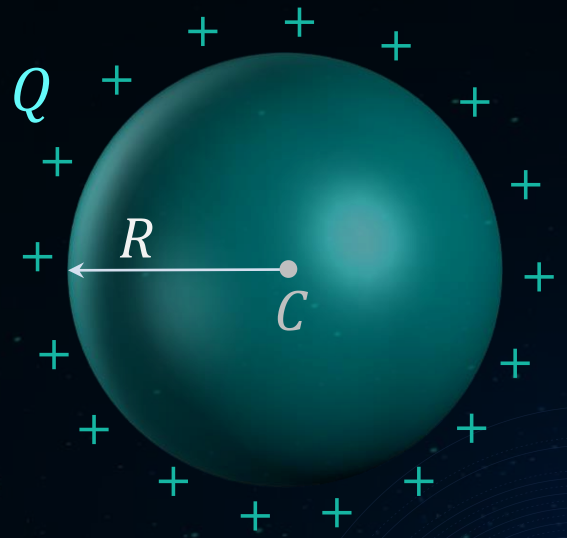
$$dW_{ext} = dq(V_f - V_i)$$

$$dw_{ext} = dq \cdot \left[\frac{kq}{R} - 0\right] = \frac{kq \cdot dq}{R}$$

U due to Hollow conducting, Solid conducting, Hollow non conducting Sphere

- Therefore, the total work required to assemble the $+Q$ charge on the sphere is,

$$W = \int dW_{ext} = \int_0^Q \frac{kq}{R} dq = \frac{kQ^2}{2R}$$



- Since the work done by the external agent is stored as the potential energy of the system, the potential energy of the sphere will be:

$$U = \frac{kQ^2}{2R}$$

- If the total charge on the sphere Q is negative, then also the potential energy of the sphere will be $\frac{kQ^2}{2R}$.

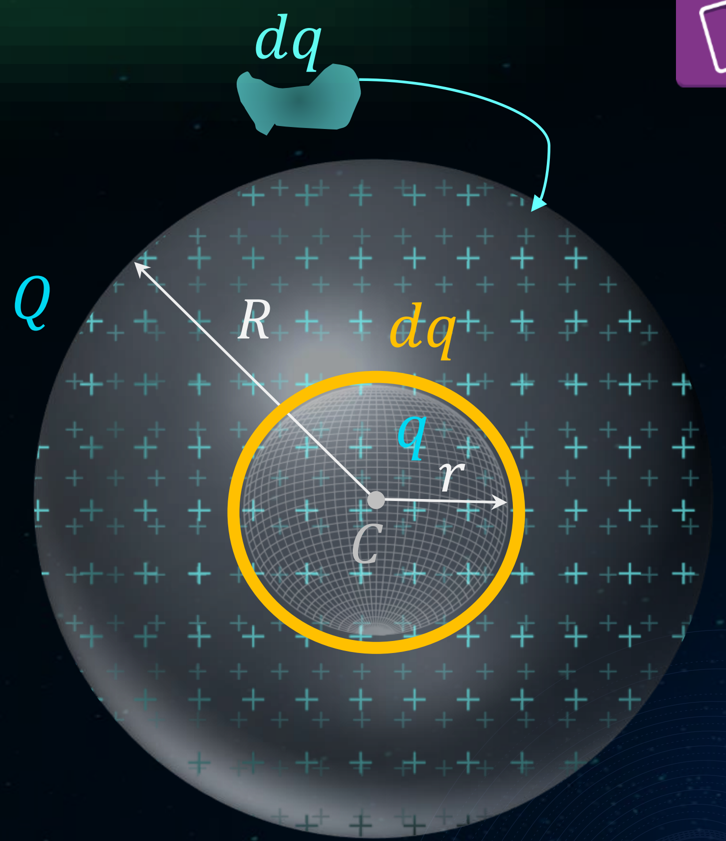
- This potential energy is stored in the electric field generated by the sphere and in this case, the electric field extend from R to ∞.

U due to Solid non conducting Sphere

- Consider a uniformly charged SNC sphere of radius R having total charge $+Q$. The charge gets distributed throughout the volume of the sphere as the sphere is non-conducting. Therefore, the volume charge density of the sphere is, $\rho = \frac{Q}{\left(\frac{4}{3}\pi R^3\right)}$

- Assume that initially we have an uncharged SNC sphere. To charge the sphere, charges have to be brought from infinity to the given configuration. Thus, consider an external agent brings a charge $+dq$ from infinity to the surface of an imaginary sphere (have charge $+q$) of radius r inside the original sphere, as shown in the figure.

  Therefore, the total charge of the sphere of radius r $(r < R)$ is,



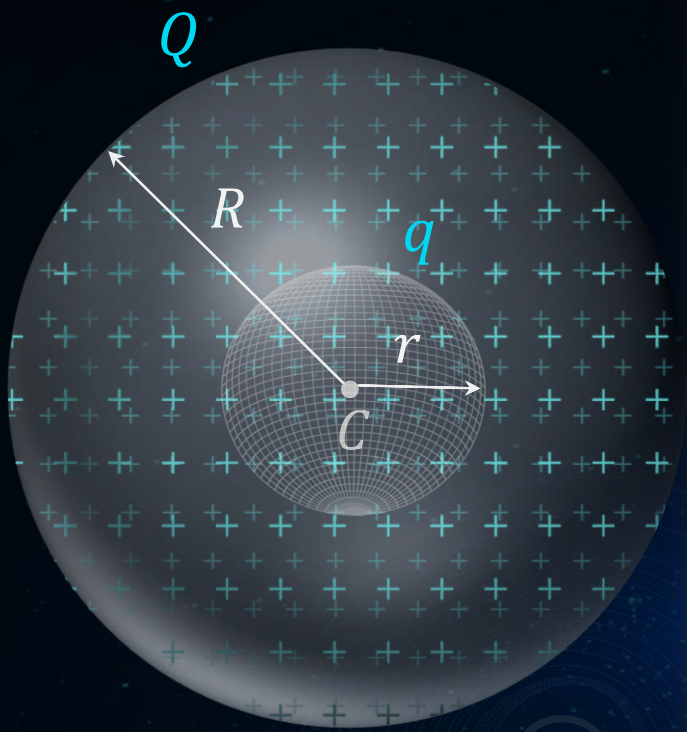
$$q = \rho\left(\frac{4}{3}\pi r^3\right) \quad \Rightarrow q = \frac{Q}{\left(\frac{4}{3}\pi R^3\right)}\left(\frac{4}{3}\pi r^3\right) \quad \Rightarrow q = \frac{Qr^3}{R^3}$$

- If thickness of the sphere having charge q and radius r gets increased by dr because of the implementation of the charge $+dq$ on the sphere, then the charge of the spherical layer will be,

$$dq = \rho(4\pi r^2 dr) \quad \Rightarrow dq = \frac{Q}{\left(\frac{4}{3}\pi R^3\right)}(4\pi r^2\, dr) \quad \Rightarrow dq = \frac{3Q}{R^3} r^2\, dr$$

U due to Solid non conducting Sphere

- The electric potential at the surface of the sphere of radius r having total charge q is, $V = \frac{kq}{r}$



- Therefore, the total work required to assemble the charges throughout the volume of the sphere is,

$$W = \int dW_{ext} = \int V dq = \int \frac{kq}{r} dq$$

(Here, q and r both are variable)

Substituting the value of q and dq, we get,

$$\Rightarrow W = \int_0^R \frac{k}{r}\left(\frac{Qr^3}{R^3}\right)\frac{3Q}{R^3} r^2\, dr$$

$$\Rightarrow W = \frac{3kQ^2}{R^6}\int_0^R r^4\, dr \quad \Rightarrow W = \frac{3kQ^2}{R^6}\left[\frac{r^5}{5}\right]_0^R \quad \Rightarrow W = \frac{3kQ^2}{5R}$$

- Since the work done by the external agent is stored as the potential energy of the system, the potential energy of the sphere will be:

$$U = \frac{3kQ^2}{5R}$$