PHYSICS

KINEMATICS

EQUATIONS OF MOTION

NOTES

What you already know

- Speed and velocity
- Acceleration

What you will learn

- Acceleration and retardation
- Equations of motion

Example

The relation between time t and distance x is $t = ax^2 + bx$, where a and b are constants. The acceleration is

(A) $-2abv^2$ (B) $2bv^3$ (C) $-2av^3$ (D) $2av^2$

Solution

Given

$t = ax^2 + bx$ ....(i)

Differentiating both the sides of the equation (i) with respect to time,

$$\Rightarrow \frac{d}{dt}(t) = \frac{d}{dt}(ax^2) + \frac{d}{dt}(bx)$$

$$\Rightarrow 1 = 2ax\left(\frac{dx}{dt}\right) + b\left(\frac{dx}{dt}\right)$$

We know that,

$$\text{velocity}(\vec{v}) = \frac{d\vec{x}}{dt}$$

$1 = 2axv + bv$ ....(ii)

$1 = (2ax + b)v$

$$v = \frac{1}{(2ax+b)} \text{.........}(iii)$$

$$\text{acceleration} = \frac{dv}{dt} = \frac{-2a\left(\frac{dx}{dt}\right)}{(2ax+b)^2}$$

We know that,

$v = \frac{dx}{dt} = \frac{1}{(2ax+b)}$, we get acceleration = $-2av^3$

Option C is correct.

Example

Acceleration of a particle in rectilinear motion is given as $a = 2t + 1$, where a is in ms^{-2}
Find its velocity and displacement as a function of time t. (Given, at $t = 0s$, $u = 2ms^{-1}$ & $x = 3m$)

Solution

We know that

$$a = \frac{dv}{dt}$$

$$\int_{v_i}^{v_f} dv = \int_{t_i}^{t_f} a dt$$

$$v_f - v_i = \int_{t_i}^{t_f} a dt \quad \text{........ (i)}$$

Given,

$$v_i = u = 2\ ms^{-1},\ v_f = v\ ms^{-1}$$

$$t_i = 0\ s,\ t_f = t\ s$$

$$a = (2t + 1)\ ms^{-2}$$

From equation (i),

$$\Rightarrow v - 2 = \int_0^t (2t+1) dt$$

$$\Rightarrow v - 2 = t^2 + t$$

$$\Rightarrow v(t) = (t^2 + t + 2) ms^{-1} \quad \text{........ (ii)}$$

We know that

$$v = \frac{dx}{dt}$$

$$\int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} v dt$$

Given,

$$x_i = 3\ m,\ x_f = x\ m$$

$$t_i = 0\ s,\ t_f = t\ s$$

From equation (ii),

$$v(t) = t^2 + t + 2$$

$$\int_3^x dx = \int_0^t (t^2 + t + 2)\ dt$$

$$x - 3 = \int_0^t t^2 dt + \int_0^t t dt + 2\int_0^t dt$$

$$x - 3 = \left[\frac{t^3}{3}\right]_0^t + \left[\frac{t^2}{2}\right]_0^t + 2[t]_0^t$$

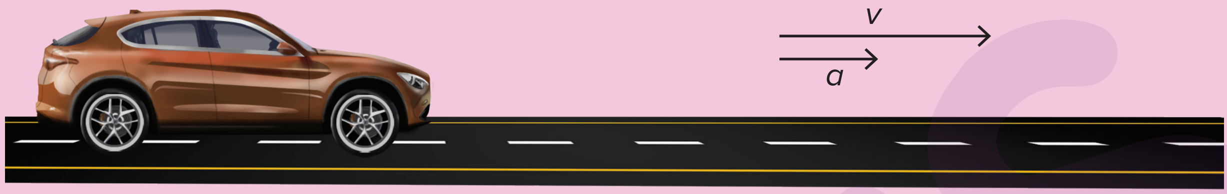
$$x - 3 = \frac{t^3}{3} + \frac{t^2}{2} + 2t$$

$$x(t) = \frac{t^3}{3} + \frac{t^2}{2} + 2t + 3$$

Acceleration

In an accelerated motion, velocity and acceleration have the same direction, resulting in an increasing velocity.

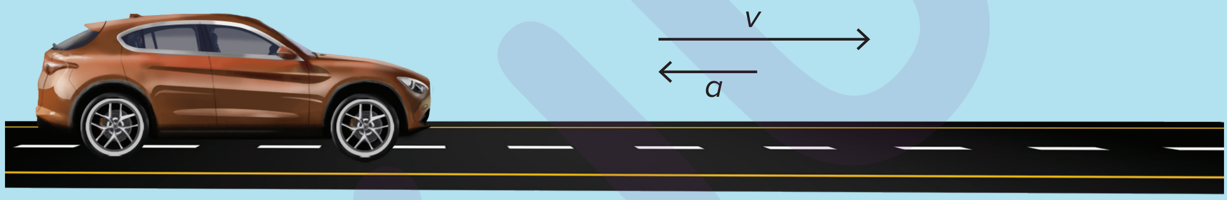
Example: In the given figure, acceleration and velocity of the car is in the same direction. Thus, the velocity of the car increases and it is an accelerated motion.



Retardation Or Deceleration

In a decelerated motion, velocity and acceleration have opposite directions, resulting in decreasing velocity.

Example - In the given figure, acceleration and velocity of the car is in the opposite direction. Thus the velocity of the car decreases and it is a retarded motion.



BOARDS

Equations of motion

Let a body, moving with a constant acceleration a, achieve velocity v at time t, with a displacement s.

Let u be the initial velocity of the body at t = 0 s

First equation of motion

We know that,

$$a = \frac{dv}{dt}$$

Rearranging the above equation

$dv = adt$

Integrating both the sides, we get,

$$\Rightarrow \int_{v_i}^{v_f} dv = \int_{t_i}^{t_f} adt$$

For constant acceleration (a = constant), we have,

$$\Rightarrow \int_{v_i}^{v_f} dv = a\int_{t_i}^{t_f} dt$$

$$\Rightarrow v_f - v_i = a\left(t_f - t_i\right) \text{........ (i)}$$

Here

$v_f = v, v_i = u, t_f = t, t_i = 0$

From equation (i),

$\Rightarrow v-u=a(t-0)$

$\Rightarrow v-u=at$

$\Rightarrow v=u+at$

Therefore, the first equation of motion is

$v = u + at$

In vector form,

$\vec{v} = \vec{u} + \vec{a}t$

where

v = Velocity at time t

u = Initial velocity

a = Acceleration

Example

The acceleration of a particle, moving in the x-direction, with initial velocity $5ms^{-1}$, is $2ms^{-2}$. Find its velocity at $t = 3\ s$.

Solution

We know,

$v=u+at$

Given $u = 5\ ms^{-1}, a = 2\ ms^{-2}, t = 3\ s$

$v = 5 + 2 \times 3$

$v = 11\ ms^{-1}$

Velocity of the particle at t = 3 s is 11 ms^{-1}

Second equation of motion

We know that,

$v=\frac{dx}{dt}$

Rearranging above equation

$dx = vdt$

From the first equation of motion, we know that $v = u + at$

Putting this value of v in the above equation, we get,

$dx = (u + at)\ dt$

Integrating both the sides of the above equation, we get,

$$\Rightarrow \int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} (u + at)\,dt$$

$$\Rightarrow x_f - x_i = u(t_f - t_i) + \frac{1}{2}a\left((t_f)^2 - (t_i)^2\right)$$

$x_f - x_i$ = Displacement of the particle = s

u = Initial velocity of the particle

$t_f = t\ s,\ t_i = 0\ s$

$$\Rightarrow s = u(t - 0) + a\left(\frac{t^2}{2} - \frac{0}{2}\right)$$

$$\Rightarrow s = ut + \frac{1}{2}at^2$$

Therefore, the second equation of motion is

$$s = ut + \frac{1}{2}at^2$$

In vector form,

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

Example

The acceleration of a particle, moving in the x-direction, with initial velocity $5ms^{-1}$, is $2ms^{-2}$. Find its displacement at $t = 3\ s$.

Solution

$$s = ut + \frac{1}{2}at^2$$

Given $u = 5ms^{-1},\ a = 2\ ms^{-2},\ t = 3s$

$$s = 5 \times 3 + \frac{1}{2} \times 2 \times 3^2$$

$$s = 15 + 9 = 24\ m$$

Therefore, displacement at $t = 3\ s$ is 24 m.

Example

The acceleration of the block moving in x-direction, with initial velocity $8ms^{-1}$, is $-2ms^{-2}$. Find

(a) The velocity of the block at $t = 5\ s$

(b) The displacement of the block in $t = 5\ s$

(c) The distance traveled by the block in 5 s

Solution

Given,

Initial velocity of block (u) = 8 ms^{-1}

Acceleration of the block (a) = -2 ms^{-2}

Time (t) = 5 s

Let v be the velocity of the block at any time t

(a)

Velocity of the block at any time t is given by,

$$v = u + at$$

$$\Rightarrow v = 8 + (-2) \times 5$$

$$\Rightarrow v = -2\ ms^{-1}$$

The velocity of block at $t = 5\ s$ is -2 ms^{-1}

(b)

Displacement of the block at any time t is given by,

$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow s = 8 \times 5 - \frac{1}{2} \times 2 \times 5^2$$

$$\Rightarrow s = 40 - 25 = 15\ m$$

(c)

$$v = u + at$$

$$\Rightarrow v(t) = (8 - 2t)ms^{-1}$$

Velocity of the particle will be zero if the particle turned back,
v(t) = 0

$$\Rightarrow 8 - 2t = 0$$

$$\Rightarrow t = 4\ s$$

Thus, between t = 0 s and t = 4 s, the particle travelled in a straight line in the forward direction.
Also, between t = 4 s and t = 5 s, the particle travelled in a straight line in the opposite direction.
Displacement of block at t = 4 s is

$$s_{t=4} = ut + \frac{1}{2}at^2$$

$$s_{t=4} = 8 \times 4 + \frac{1}{2} \times (-2) \times 4^2$$

$$s_{t=4} = 32 - 16 = 16\ m$$

Distance travelled by the block in 4 seconds = (Displacement at t = 4s) - (Displacement at t = 0s) = 16 - 0 = 16 m.
Also, displacement of the block at t = 5s is

$$s_{t=5} = 8 \times 5 + \frac{1}{2} \times (-2) \times 5^2 = 15\ m$$

Distance travelled by the block in fifth second = (Displacement at t = 5s) - (Displacement at t = 4s)
⇒ Distance travelled by the block in fifth second = |15 - 16| = 1m.
Total distance travelled in 5 seconds = (Distance travelled in 4 seconds) + (Distance travelled in the fifth second) = 16 + 1 = 17 m