# RELATIVE MOTION IN ONE DIMENSION

## INTRODUCTION TO RELATIVE MOTION



### What you already know

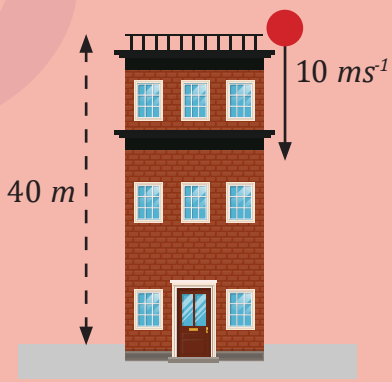
- Vectors
- Kinematics
- Math tools
- Units and dimensions

### What you will learn

- Relative motion
- Problems on relative motion

### Example

A ball is thrown vertically downward from a tower of height $40\ m$ with a speed of $10\ ms^{-1}$. Draw *s-t*, *v-t*, and *a-t* curves of the ball.




### Solution

Taking the downward direction as negative,

Given,

Height of the tower = $40\ m$

Speed of the ball = $10\ ms^{-1}$

By using the second kinematic equation, we get the following:

$$s = ut + \frac{1}{2}at^2$$

$$-40 = -10t + \frac{1}{2}(-g)t^2$$

$$-40 = -10t - 5t^2$$

$$5t^2 + 10t - 40 = 0$$

$$t^2 + 2t - 8 = 0$$

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$t^2 + 4t - 2t - 8 = 0$

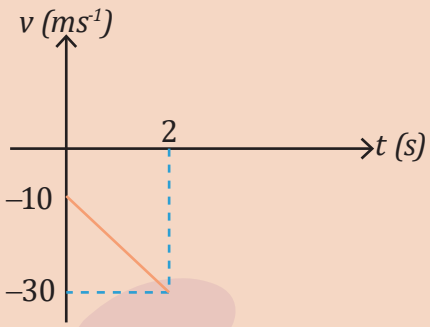
$t(t + 4) - 2(t + 4) = 0$

$(t - 2)(t + 4) = 0$

$t = 2\ s$ or $t = -4\ s$

By neglecting the negative value of time, we get the following:

Time of flight = $2\ s$




**v-t graph**

By using the first kinematic equation, we get the following:

$v = u + at$

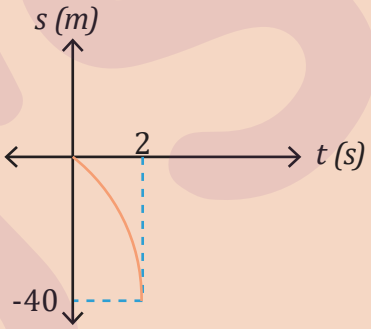
$v = -10 - 10t$ ..... *(i)*

By putting $t = 2\ s$ in equation *(i)*, we get the following:

$v = -10 - 10 \times 2$

$v = -10 - 20 = -30\ ms^{-1}$.....*(ii)*

From equations *(i)* and *(ii)*, we get the velocity vs time graph as shown in the respective figure.




**s-t graph**

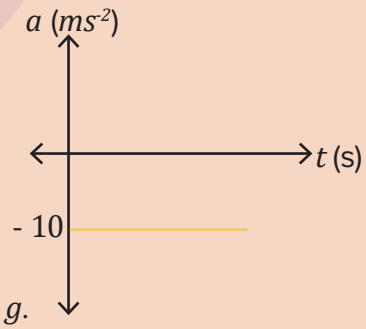
By using the second kinematic equation, we get the following:

$s = ut + \frac{1}{2}at^2$

$s = -10t - \frac{1}{2} \times 10 \times t^2$

$s = -10t - 5t^2$.....*(iii)*

From equation *(iii)*, we get the displacement vs time graph as shown in the respective figure.

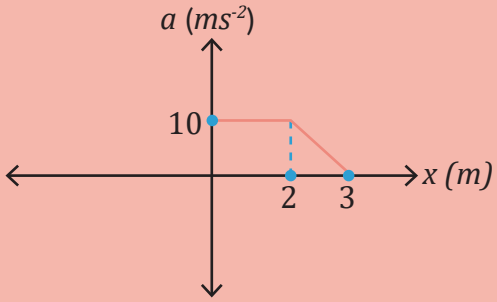



**a-t graph**

Acceleration of the ball is constant throughout the time of flight, that is - $g$.

## Example

The speed of a particle at $x = 0\ m$ is $u = 2\ ms^{-1}$. Find its speed at $x = 3\ m$.




## Solution

Given,

At $x = 0\ m$, speed of the particle, $u = 2\ ms^{-1}$

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We know,

$a = \frac{dv}{dt} = v\frac{dv}{dx}$

Area under the $a$-$x$ graph $= \int_{x_i}^{x_f} a\,dx = \int_{u}^{v} v\,dv = \left[\frac{v^2}{2}\right]_u^v = \frac{v^2 - u^2}{2}$

Let $v$ be the speed of the particle at $x = 3\ m$.

Area under the $a$–$x$ graph between $x = 0\ m$ and $x = 3\ m = \frac{v^2 - u^2}{2}$

Also, by geometry, area under the $a$-$x$ graph between $x = 0\ m$ and $x = 3\ m$

$= (10 \times 2) + \frac{1}{2} \times (3 - 2) \times 10 = 25$

Therefore,

$\frac{v^2 - u^2}{2} = 25$

$\frac{v^2 - 2^2}{2} = 25$

$v^2 = 54$

$v = \sqrt{54}\ ms^{-1}$

Hence, the velocity of the particle at $x = 3\ m$ is $\sqrt{54}\ ms^{-1}$.

MAIN

## Relative Motion in One Dimension

Let a bus be travelling with a speed of $10\ ms^{-1}$ as shown in the figure.

A boy is walking on the ground with speed of $4\ ms^{-1}$ observes a girl walking in the bus in the opposite direction with respect to bus with speed of $2\ ms^{-1}$.

Also, a boy and another girl, who are seated in the bus, observe the girl who is walking in the bus.

By taking the rightward direction as positive $x$-axis, we get the following:

Velocity of the bus, $\vec{v}_B = 10\,\hat{i}\ ms^{-1}$

Velocity of the boy outside the bus, $\vec{v}_b = 4\,\hat{i}\ ms^{-1}$

Velocity of the girl in the bus with respect to the bus, $\vec{v}_{gB} = -2\,\hat{i}\ ms^{-1}$

### Considering the ground frame

Velocity of the bus with respect to the ground $= \vec{v}_{BG} = \vec{v}_B - \vec{v}_G = 10\,\hat{i}\ ms^{-1} - 0\,\hat{i}\ ms^{-1} = 10\,\hat{i}\ ms^{-1}$

Velocity of the boy outside the bus with respect to the ground $= \vec{v}_{bG} = \vec{v}_b - \vec{v}_G = 4\,\hat{i}\ ms^{-1} - 0\,\hat{i}\ ms^{-1}$

$= 4\,\hat{i}\ ms^{-1}$

### Considering the bus frame

Velocity of the girl inside the bus with respect to the bus $= \vec{v}_{gB}$

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As we know,

$\vec{V}_{AB} = \vec{V}_{AG} - \vec{V}_{BG}$

$\vec{V}_{gB} = \vec{V}_{gG} - \vec{V}_{BG}$

$-2\,\hat{i}\,ms^{-1} = \vec{V}_{gG} - 10\,\hat{i}\,ms^{-1} \Rightarrow \vec{V}_{gG} = 8\,\hat{i}\,ms^{-1}$

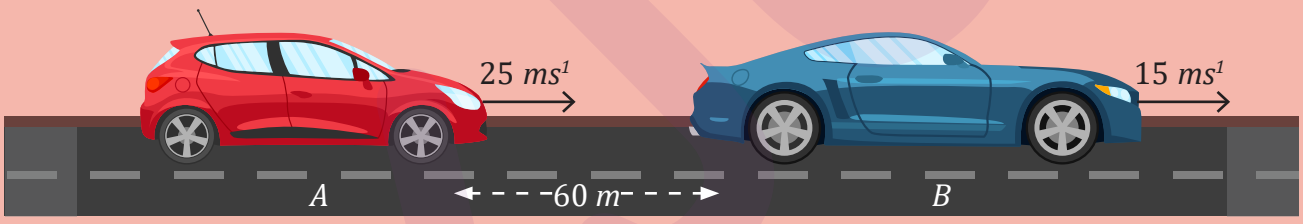
Therefore, velocity of the girl with respect to the ground = $\vec{V}_{gG} = 8\ ms^{-1}$

**From this example, we can say the following:**

- The frame of reference is necessary if we are discussing relative motion.
- The change in the frame of reference has no effect on the distance between the two points or particles and the time or individual force.

## Example

Car $A$ is moving on a straight road with a speed of 25 $ms^{-1}$. Car $B$ is also moving along the same direction with a speed of 15 $ms^{-1}$. The initial distance between the car $A$ and car $B$ is 60 $m$. Will car $A$ meet car $B$ ? If yes, then find the time and position when car $A$ overtakes car $B$.




## Solution

Taking the rightward direction as a positive $x$-axis.

Given,

Speed of car $A$ = 25 $ms^{-1}$

Speed of car $B$ = 15 $ms^{-1}$

The initial distance between the car $A$ and car $B$ is 60 $m$.

**Considering the ground frame**

Velocity of car $A$ = 25 $\hat{i}\,ms^{-1}$

Velocity of car $B$ = 15 $\hat{i}\,ms^{-1}$

As the speed of car $A$ is more than the speed of car $B$, car $A$ will meet car $B$.

Let, after time $t$ car $A$ overtakes car $B$.

Let us take car $A$'s initial position as the origin.

Initial distance between car $A$ and car $B$ is 60 $m$.

Therefore,

Initial position of car $B$ is $x$ = 60 $m$

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Displacement of car $A$ in time $t = 25t$.....*(i)*

Displacement of car $B$ in time $t = 15t$.....*(ii)*

We know,

$x_f - x_i$ = Displacement

From equation *(i)*, we get the following:

$x_f - 0 = 25t$

$x_f = 25t$

Therefore, the final position of car $A$ is $25t$.

From equation *(ii)*, we get the following:

$x_f - 60 = 15t$

$x_f = 15t + 60$

Therefore, the final position of car $B$ is $60 + 15t$.

At time $t$, both the cars are at the same position where they meet each other.

Therefore,

$15t + 60 = 25t$

$10t = 60$

$t = 6\ s$

Hence, car $A$ and car $B$ meet at $t = 6\ s$.

**Alternative method**

Considering the car $B$ frame,

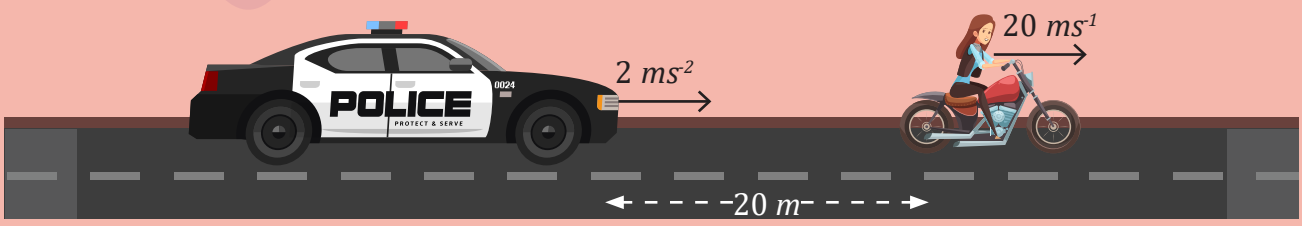
Velocity of car $A$ with respect to car $B$, $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B \Rightarrow \vec{v}_{AB} = 25\ \hat{i}\ ms^{-1} - 15\ \hat{i}\ ms^{-1} = 10\ \hat{i}\ ms^{-1}$.....*(iii)*

Distance between car $A$ and car $B = 60\ m$

Time taken by car $A$ to meet car $B = \frac{60\ m}{10\ ms^{-1}} = 6\ s$

**Example**

A thief is running away on a straight road on a bike moving with a speed of 20 $ms^{-1}$. A police car, initially at the rest, starts chasing the thief with an acceleration of 2 $ms^{-2}$. The initial distance between the thief and the police is 20 $m$. Find the time taken by the police to catch the thief.

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MAIN

**Solution**

Taking the rightward direction as positive $x$-axis,

Given,

Speed of the thief = 20 $ms^{-1}$

Acceleration of the police car = 2 $ms^{-2}$

Initial distance between the thief and the police = 20 $m$

**Considering the ground frame**

Velocity of the thief with respect to the ground = $\vec{v}_{TG} = \vec{v}_T - \vec{v}_G = 20\ \hat{i}\ ms^{-1} - 0\ \hat{i}\ ms^{-1} = 20\ \hat{i}\ ms^{-1}$

Initial velocity of the police car with respect to the ground = $0\ \hat{i}\ ms^{-1}$

Acceleration of the police car with respect to the ground

$\vec{a}_{PG} = \vec{a}_P - \vec{a}_G = 2\hat{i} - 0 = 2\hat{i}\ ms^{-2}$

**Considering the thief frame**

Initially velocity of the police car with respect to the thief = $\vec{u}_{PT} = \vec{u}_{PG} - \vec{u}_{TG}$

= $0\ \hat{i}\ ms^{-1} - 20\ \hat{i}\ ms^{-1} = -20\ \hat{i}\ ms^{-1}$

Acceleration of the police car with respect to the thief = $\vec{a}_{PT} = \vec{a}_{PG} - \vec{a}_{TG} = 2\ \hat{i}\ ms^{-2}$

Displacement of the police car with respect to the thief = $20\ \hat{i}\ m$

By using the second kinematic equation, we get the following:

$s = ut + \frac{1}{2}at^2$

$20 = -20t + \frac{1}{2} \times 2t^2$

$20 = -20t + t^2$

$t^2 - 20t - 20 = 0$

$t = \frac{-(-20) \pm \sqrt{(-20)^2 - 4 \times 1 \times (-20)}}{2 \times 1}$

$t = 10 \pm 2\sqrt{30}\ s$

Ignoring, $t = 10 - 2\sqrt{30}\ s$ , since it is negative, and time cannot be negative.

Therefore, time taken by the police to catch the thief is $10 + 2\sqrt{30}\ s$.

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