Newton's Second Law of Motion

It states that the rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts.

Mathematical formulation of Newton's Second Law of Motion:

Let mass of a moving object be m.
Let is initial velocity be u and final velocity be v.
We know that momentum (p) = Mass × velocity
Therefore,
Initial momentum of object = mu
And Final momentum of the object = mv
Therefore, change in momentum = mv – mu

$\Rightarrow$ Rate of change of mometum $= \frac{\text{mv} - \text{mu}}{\text{t}} = \frac{\text{m(v} - \text{u)}}{\text{t}}$

Now, from the Newton's 2^{nd} Law of Motion, we have:
Force $\propto$ Rate of change of momentum

$$F \propto \frac{mv - mu}{t}$$

$$\Rightarrow F \propto \frac{m(v - u)}{t} \qquad \dots (i)$$

But we know that $\left(\frac{v - u}{t}\right) = a$ (Acceleration)

Using, above relation in equation (i), we get

$F \propto ma$

$F = kma \qquad \dots(ii)$

Where k is the proportionality constant

Now, 1 unit force is defined as the force applied on an object of mass 1kg to produce the acceleration of $1 m/s^2$.

Thus, 1 unit of force $= k \times 1kg \times 1m/s^2$

$\Rightarrow k = 1$

By putting the value of k = 1 in equation (ii), we get:

$F = ma$

i.e., Force = Mass × Acceleration

The SI unit of Force

SI unit of force is Newton (N).
Since Force = Mass x Acceleration
The unit of mass = kg and The unit of acceleration = m/s^2
If force, mass and acceleration is taken as 1 unit.
Therefore,

1 Newton (N) = 1kg x $1 m/s^2$

Thus, Newton (N) = kg m/s^2

Thus, one unit of force is defined as the amount that produces an acceleration of 1 m/s^2 in an object of mass 1 kg.

Applications of Newton's 2nd Law of Motion

- A fielder pulls his hand backward; while catching a cricket ball coming with a great speed. Actually, while catching a cricket ball the momentum of ball is reduced to zero. If the ball is stopped suddenly, its momentum will be reduced to zero instantly causing the instant rate of change in momentum due to which ball will exert great force on the hands of player due to which the player's hand may get injured. Therefore, by pulling the hand backward a fielder gives more time to the change of momentum to become zero. This prevents the hands of fielder from getting hurt.
- For athletes of long and high jump, sand bed or cushioned bed is provided at the place of landing. This is because when an athlete falls on the ground after performing a high or long jump, the momentum of his body is reduced to zero. If the momentum of an athlete will be reduced to zero instantly, it will result in the production of a large force which may hurt the player. Whereas, by providing a cushioned bed, the momentum of player's body is reduced to zero in a delayed period due to which less force acts on his body hence, preventing the athlete from getting hurt.
- Seat belts in a car are provided to prevent the passenger from getting thrown in the direction of motion. In case of sudden braking or any accident, passengers may get thrown in the direction of motion of vehicle and may get fatal injuries. Whereas, the stretchable seat belts prevent the passenger's body to fall suddenly and thus increase the time of the rate of momentum to be become zero. This will reduce the effective force hence preventing the passenger from getting any fatal injury.
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Newton's Third Law of Motion

Newton's Third Law of Motion states that there is always reaction for every action in opposite direction and of equal magnitude, i.e., action and reaction forces are equal and opposite.

Applications of Newton's Third Law of Motion:

- Recoil of gun: When bullet is fired from a gun, it moves ahead. By the Newton's 3^{rd} law of motion, the bullet apply same force on gun in backward direction. Due to this force, gun moves back giving a jerk to the shoulder of the gunman. This is called recoil of gun. Here, gun moves back only by small amount due to its heavy mass.

- Walking of a person: A person is able to walk due to the Newton's Third Law of Motion. During walking, a person pushes the ground in backward direction and in the reaction the ground also pushes the person with equal magnitude of force but in opposite direction. This enables him to move in forward direction against the push.
- Swimming in water: Man pushes water back by applying force. By Newton's 3 Law, water applies equal and opposite force on swimmer. Due to this force man moves ahead.
- Propulsion of a boat in forward direction – Sailor pushes water with oar in backward direction; resulting water pushing the oar in forward direction. Consequently, the boat is pushed in forward direction.

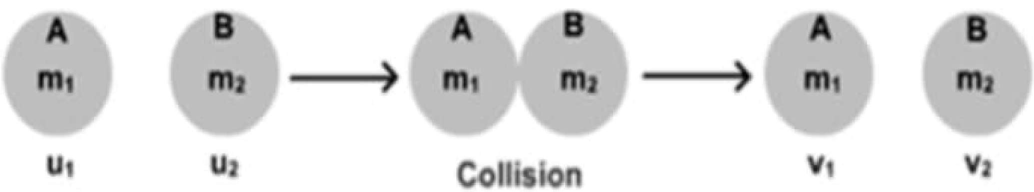
Conservation of Momentum

If t or more objects apply force on each other with no external force, their final momentum remains same as initial momentum.

Total momentum before collision = Total momentum after collision

Mathematical Formulation of Conservation of Momentum:

Suppose, two objects A and B each of mass m_1 and mass m_2 are moving initially with velocities u_1 and u_2, strike each other after time t and start moving with velocities v_1 and v_2 respectively.



We know that, Momentum = Mass x Velocity

Therefore,

Initial momentum of object A = m_1u_1

Initial momentum of object B = m_2u_2

Final momentum of object A = m_1v_1

Final momentum of object B = m_2v_2

Now, Rate of change of momentum = Change in momentum/ time taken

Therefore,

$$F_{AB} = \frac{(m_1v_1 - m_1u_1)}{t}$$

$$\Rightarrow F_{AB} = \frac{m_1(v_1 - u_1)}{t} \qquad ...(i)$$

Also, Rate of change of momentum in B during collision,

$$F_{BA} = \frac{(m_2v_2 - m_2u_2)}{t}$$

$$\Rightarrow F_{BA} = \frac{m_2(v_2 - u_2)}{t} \qquad \text{... (ii)}$$

But, from Newton's third law of motion, we have:

$$F_{AB} = -F_{BA}$$

$$\Rightarrow \frac{m_1(v_1 - u_1)}{t} = -\frac{m_2(v_2 - u_2)}{t}$$

$$\Rightarrow m_1v_1 - m_1u_1 = -m_2v_2 + m_2u_2$$

$$\Rightarrow m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$

Thus, Total initial momentum = Total final momentum

Applications of Conservation of Momentum:

- Propelling of rocket: The chemicals inside the rocket burn and produce the high velocity blast of hot gases. These gases get ejected downwards with a great velocity. To conserve the total momentum of gases, the rocket moves up with a large velocity.
- Flight of jet planes: In jet planes, a large volume of gases produced by combustion of fuel is allowed to escape through a jet in backward direction. Due to the high velocity, the backward moving gases have a large momentum. In order to conserve the momentum, the plane get a push in forward direction and moves with the great speed.